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NON(ANTI)COMMUTATIVE SYM THEORY: RENORMALIZATION IN SUPERSPACE

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ABSTRACT

We present a systematic investigation of one–loop renormalizability for nonanticommutative $N = 1/2$, $U(\mathcal{N})$ SYM theory in superspace. We first discuss classical gauge invariance of the pure gauge theory and show that in contradistinction to the ordinary anticommutative case, different representations of supercovariant derivatives and field strengths do not lead to equivalent descriptions of the theory. Subsequently we develop background field methods which allow us to compute a manifestly covariant gauge effective action. One–loop evaluation of divergent contributions reveals that the theory simply obtained from the ordinary one by trading products for star products is not renormalizable. In the case of SYM with no matter we present a $N = 1/2$ improved action which we show to be one–loop renormalizable and which is perfectly compatible with the algebraic structure of the star product. For this action we compute the beta functions. A brief discussion on the inclusion of chiral matter is also presented.

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1 Introduction and Conclusions

In the recent past the study of field theories defined on noncommutative spaces has received new impetus from the realization that, in a string theory context, the low-energy dynamics of D-branes is affected by the presence of world-volume fluxes in a manner which can be described by such field theories [1]-[5]. The effect of noncommutativity manifests itself in the appearance of nontrivial product properties: the multiplication of fields is no longer commutative but described by a so-called $*$-product. Similarly, when the basic setup is supersymmetric, the description in superspace of the corresponding superfield theories is by means of modified anticommutation relations for the spinor coordinates. In the simplest case, that of a constant graviphoton flux $\mathcal{F}_{\alpha\beta}$, the superspace geometry is modified through a nontrivial anticommutator $\{\theta^\alpha, \theta^\beta\} = \mathcal{F}^{\alpha\beta}$ instead of the usual vanishing one. (Since one keeps the anticommutator of the conjugate variables at zero, $\{\overline{\theta}^{\dot{\alpha}}, \overline{\theta}^{\dot{\beta}}\} = 0$, it is necessary to work in Euclidean space where dotted and undotted spinors are unrelated.) Again, the product of superfields is now a $*$-product. The nontrivial anticommutativity (NAC) rule usually leads to a deformation of SUSY which corresponds to partial breaking of supersymmetry. This manifests itself by the appearance, in the component actions, of additional couplings with reduced invariance properties. For example, $N = 1$ global SUSY is broken down to $N = 1/2$ with supersymmetry broken in the antichiral sector.

The study of the NAC supersymmetric geometry was initiated in a number of papers [6, 7, 8] and the corresponding description of the resulting field theories first given by Seiberg [2] and subsequently by a number of other authors. A partial list consists of, in components, refs. [9]-[13] and in superspace, refs. [14]-[25] (it should be emphasized that the starting point is always the superfield description with a nontrivial $*$-product and the subsequent decomposition into components). Of interest has been the question of their renormalizability. In the absence of deformations the high-energy behavior of SUSY theories is very much softened by the standard boson-fermion cancellations; to what extent does this behavior survive the NAC deformation?

The renormalizability issue has been studied in components, and, by the present authors, in superspace. We have considered both deformed WZ and SYM models. For the former case it was possible to give a complete answer: already at the one–loop level renormalizability is lost but by adding a single new coupling in the classical theory, dependent on the deformation parameter, it can be restored not only at one loop but to all orders of perturbation theory [10, 18]. Divergences are still only logarithmic so that the induced SUSY breaking is soft. This follows also from the fact that the NAC deformation can be mimicked by means of a spurion field. The same conclusions were reached in components [11].

The situation for NAC SYM is more complicated. It is straightforward to start with a superspace description $S \sim \int \mathrm{Tr}(W^\alpha * W_\alpha)$ where the W's themselves contain implicit

star products and go to components (although a fermion field shift is necessary in order to maintain the usual component transformation rules), but such an approach requires choosing a WZ gauge *ab initio* and leaves open the question whether in the presence of the star product something has been lost. Nonetheless, proceeding with the component approach, a number of results have been obtained. In particular Jack, Jones and Worthy [12] have provided the most complete results of the one–loop quantum properties of the deformed component theory and shown that by adding new, deformation-parameter-dependent terms to the classical theory the one–loop effective action can be renormalized. In the meantime we had examined the superspace situation. At the one–loop level, in a background field approach (which maintains control on the gauge invariance of the theory) it was shown [21] that new divergences were indeed present which could not be removed by renormalization. The important point however was the check that indeed, even in the presence of NAC, the effective action is gauge invariant and therefore the safety of going to WZ gauge is ensured.

We have continued the superspace work and in this paper we show how, by again adding new terms to the classical action, we can make the superspace theory one-loop renormalizable, generally confirming the work of [12]. We prove that subtraction of one–loop divergences does not require renormalization of the NAC parameter. Therefore, renormalization does not deform the star product. We have also been able to show that the supersymmetry breaking due to the NAC geometry is soft. In the rest of this Section we give a detailed summary of our procedure and results and, to save the reader from leafing to the end of this rather technical report, we present also our conclusions.

The classical action

SYM is best described in a covariant approach by means of covariant derivatives and field strengths satisfying a number of constraints. For quantization purposes these constraints have to be solved in terms of unconstrained superfields, and this role is played by the gauge prepotential $V = V^A T_A$ where T_A are the generators of the gauge group $U(\mathcal{N})$. (In the ordinary theory it would be sufficient to study $SU(\mathcal{N})$ and $U(1)$ separately but, as we shall see, in the NAC case the two subgroups are intimately linked and must be considered together). The field strength W_α (and its conjugate) is expressed in terms of exponentials of the prepotential and their spinor derivatives. Of course everything involves the star product.

Whereas in Minkowski space V is usually considered real, it turns out that in order to maintain certain conjugation properties of the (spinor) covariant derivatives it is necessary to choose V imaginary. In both the ordinary and NAC case the description in terms of V introduces a certain asymmetry between some of the geometric quantities and their conjugates, and one has the choice of *gauge chiral representation* or *gauge antichiral representation*. In the first case the prepotential covariantizes only the chiral spinor derivative D_α while the antichiral derivative $\overline{D}_{\dot\alpha}$ needs not be covariantized. In the

second case the opposite is true. The corresponding field strengths also differ slightly in their prepotential dependence. We denote them as W_α , $\widetilde{\overline{W}}_{\dot{\alpha}}$ and $\widetilde{W}_\alpha$, $\overline{W}_{\dot{\alpha}}$. The choice of representation makes no difference in the usual, AC, case. It turns out that in the NAC case, and especially in the background field method, this choice is not totally arbitrary.

In the AC case, and in the absence of instantons, the choice among four possible actions $S \sim \int W^\alpha W_\alpha$, its complex conjugate, or the corresponding antichiral gauge quantities has no consequences; they all lead to the same component action. This is no longer true in the NAC case. Whereas in the usual case the equivalence is established by making use of cyclicity of the trace in quantities such as, for example, $\int d^2\overline{\theta}\mathrm{Tr}(e^{-V}\overline{W}^{\dot{\alpha}}\overline{W}_{\dot{\alpha}}e^V)$, the cyclicity is lost unless integration over $d^2\theta$ is present. Even gauge invariance can be lost. Our preferred choice, before quantization can proceed, is for the action

$$S = \frac{1}{2g^2}\int d^4x d^2\overline{\theta}\ \mathrm{Tr}(\overline{W}^{\dot{\alpha}}\overline{W}_{\dot{\alpha}}) \tag{1.1}$$

in antichiral representation.

The U(1) problem

In the NAC case $U(1)$ is interlinked with $SU(\mathcal{N})$, especially in the transformation properties of the corresponding gauge fields. In particular, the $U(1)$ field transforms nontrivially under the $SU(\mathcal{N})$ group. Consequently, although the action (1.1) which contains both gauge fields is invariant under the full group, a separate action

$$S_0 = \frac{1}{2g_0^2}\int d^4x d^2\overline{\theta}\ \mathrm{Tr}(\overline{W}^{\dot{\alpha}})\mathrm{Tr}(\overline{W}_{\dot{\alpha}}) \tag{1.2}$$

which describes just the $U(1)$ field (and which is also generated by quantum corrections) is not. Only by completing this action with an additional piece involving the cubic term $\mathcal{F}^{\rho\gamma}\mathrm{Tr}\left(\partial_{\rho\dot{\rho}}\overline{\Gamma}^{\dot{\alpha}}\right)\mathrm{Tr}\left(\overline{W}_{\dot{\alpha}}\overline{\Gamma}_\gamma{}^{\dot{\rho}}\right)$ we can maintain gauge invariance. Furthermore, as we shall see, the generation of this additional piece and the quadratic part thereof complicates the choice of a suitable gauge-fixing term; it forces us to a propagator choice away from Feynman gauge, thus leading to more cumbersome supergraph evaluation.

The background field method

The computation of the effective action is best done using the background field method since gauge invariance of the resulting effective action is maintained (although, in principle problems could arise in the NAC case). Interestingly, unlike the AC case, as a consequence of V being pure imaginary, it is possible to perform a background-quantum splitting, as we did, where *all* quantities, quantum, background and total, can be chosen in antichiral

representation, in contradistinction to the usual, Minkowski, case where the background has to be in vector representation.

The splitting can be performed in a manner analogous to the usual case, with $\nabla_\alpha = \boldsymbol{\nabla}_\alpha = D_\alpha$, $\nabla_{\dot\alpha} = e_*^V * \boldsymbol{\nabla}_{\dot\alpha} * e_*^{-V}$ for the derivatives and $\overline{\Phi} = \overline{\boldsymbol{\Phi}}$, $\Phi = e_*^V * \boldsymbol{\Phi} * e_*^{-V}$ for chiral superfields, where plain and boldface letters indicate quantum and background quantities, respectively; and one-loop Feynman rules can be derived as in the AC case [26].

A slight complication manifests itself in computing matter or ghost contributions to the effective action. Using the doubling trick explained in that reference one separates contributions from only chiral superfields and from antichiral superfields, but using the fact that these superfields are complex conjugates of each other it is easy to see that these contributions are equal. In the NAC case chiral and antichiral fields are not related by complex conjugation. Nonetheless, by explicit examination of the manner in which one–loop contributions are obtained, it is possible to show that they still contribute equally and only one of them need be computed explicitly. After the splitting, the evaluation of contributions from the gauge fields themselves presents no new problem. Their quantization proceeds in straightforward manner (except for the complications with the $U(1)$ fields noted above).

Structure of the divergent terms

Before computing the one-loop divergences it is useful to fix the general structure of local terms allowed by gauge invariance and other symmetries. Knowing this is a useful guide to the subsequent calculation and allows us to eliminate from the beginning diagrams which would not lead to the appropriate structures.

Gauge invariance of the background field action implies that allowed terms can only depend on background field strengths and gauge connections but not on the background prepotential. They will also depend on the deformation parameter. Furthermore, if written as integrals over full superspace, they may also depend explicitly on $\overline{\theta}^{\dot\alpha}$ as a consequence of the explicit breaking of $N = 1$ SUSY. As in other recent studies [11, 19, 18] it is also helpful to take advantage of additional global (pseudo)symmetries of the classical action which lead to further restrictions (note that possible anomalies would not affect these symmetries of the local terms). In the present case we do have one global R–symmetry and we can assign specific R–weights to the various quantities that can appear in the effective action. The resulting divergent structures (we employ a cutoff Λ for convenience and also use dimensional arguments), are quite limited in number and can be classified prior to any calculations.

One-loop divergences and lack of renormalizability

We have found convenient to perform perturbative calculations in momentum superspace by Fourier transforming all the superspace variables and in particular introducing spinorial momenta π_α conjugate to the derivatives ∂_α. In this setup the presence of the star product is signaled by the appearance at the vertices of phase factors of the form $\exp[\pi_\alpha \mathcal{F}^{\alpha\beta}\pi_\beta]$. D–algebra rules now require that in the integration over spinorial loop momenta exactly one π^2 and one $\overline{\pi}^2$ be left for each loop. It is important to note that, while in the planar diagrams the resulting phase factor does not depend on the loop momenta and powers of π and $\overline{\pi}$ only come from propagators and vertices, in the nonplanar ones powers of π come also from the expansion of the nontrivial phase factor and new divergent contributions proportional to the NAC parameter arise. This is the way NAC geometry affects the UV properties of our theory.

We have computed the divergences of the gauge effective action stemming from vector, as well as the (chiral) matter and ghost loops. We find contributions from planar diagrams (these are of course the standard, renormalizable ones), as well as ones proportional to the deformation parameter and its square coming from nonplanar diagrams. In the background field method none of these arise from the vector loops themselves. They are proportional to

$$\begin{aligned}\Gamma^{(1)}_{gauge} \quad \rightarrow \quad \frac{1}{2}(-3+N_f) \times \Big\{ \int d^4x\ d^4\theta\ \Big[\mathcal{N}\mathrm{Tr}\left(\overline{\Gamma}^{\dot{\alpha}}\ \overline{W}_{\dot{\alpha}}\right) - \mathrm{Tr}\left(\overline{\Gamma}^{\dot{\alpha}}\right)\mathrm{Tr}\left(\overline{W}_{\dot{\alpha}}\right)\Big] \\ -4i\mathcal{F}^{\rho\gamma}\int d^4x\ d^4\theta\ \ \overline{\theta}^2\ \mathrm{Tr}\left(\partial_{\rho\dot{\rho}}\overline{\Gamma}^{\dot{\alpha}}\right)\mathrm{Tr}\left(\overline{W}_{\dot{\alpha}}\overline{\Gamma}_{\gamma}^{\ \dot{\rho}}\right) \\ +\mathcal{F}^2\int d^4x\ d^4\theta\ \ \overline{\theta}^2\ \mathrm{Tr}\left(\overline{\Gamma}^{\dot{\alpha}}\overline{W}_{\dot{\alpha}}\right)\mathrm{Tr}\left(\overline{W}^{\dot{\beta}}\overline{W}_{\dot{\beta}}\right)\Big\} \qquad (1.3)\end{aligned}$$

As shown in [21], whereas the first and last terms are separately gauge-invariant, gauge invariance of the second and third terms is true only when they appear in this particular combination.

The resulting divergences cannot be renormalized away. This can be seen at the superspace level, but it can also be verified by going to components, in agreement with [12].

Renormalizable deformations and beta-functions

As was the case in the Wess-Zumino model, and is also the case in the component discussions of NAC SYM, it is possible to deform the classical action in such a way as to produce a one–loop renormalizable theory. The manner in which we have proceeded is to start *ab initio* with a deformed action containing all possible terms allowed by gauge invariance, R–symmetry, and dimensional considerations. As mentioned earlier, one of the complications that arise in performing the calculations is the appearance of a separate $U(1)$ term with, a priori, a different weight from its gauge invariant combination partner; the simplest way to proceed, as we did, is to accept the fact that Feynman gauge for this

field is ultimately not a simple choice and we must use a more complicated propagator. Aside from this, the calculations proceed apace.

We find that in the presence of new "classical" terms, vector loops themselves contribute now to the one-loop divergences. Once they are all calculated we end up with an obviously one-loop renormalizable situation depending on a number of arbitrary coupling constants. We then compute the β-functions and we find that at this one-loop level they allow for specific restrictions on these constants. In particular, there are two choices which lead to minimal deformed actions which are one-loop renormalizable:

$$\begin{aligned}
S_{min} = &\frac{1}{2\,g^2}\int d^4x\ d^4\theta\ \mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\ \overline{W}_{\dot\alpha}\right)\\
&+\frac{1}{2\,g_0^2\,\mathcal{N}}\int d^4x\ d^4\theta\ \Big[\ \mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\right)\mathrm{Tr}\left(\overline{W}_{\dot\alpha}\right)\\
&\qquad\qquad +4i\mathcal{F}^{\rho\gamma}\overline{\theta}^2\ \mathrm{Tr}\left(\partial_{\rho\dot\rho}\overline{\Gamma}^{\dot\alpha}\right)\mathrm{Tr}\left(\overline{W}_{\dot\alpha}\overline{\Gamma}_{\gamma}{}^{\dot\rho}\right)\\
&\qquad\qquad -\mathcal{F}^2\overline{\theta}^2\ \mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\overline{W}_{\dot\alpha}\right)\mathrm{Tr}\left(\overline{W}^{\dot\beta}\overline{W}_{\dot\beta}\right)\ \Big]\\
&+\frac{1}{l^2}\,\mathcal{F}^2\int d^4x\ d^4\theta\ \overline{\theta}^2\ \mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\ \overline{W}_{\dot\alpha}\ \overline{W}^{\dot\beta}\ \overline{W}_{\dot\beta}\right)
\end{aligned}\tag{1.4}$$

or

$$\begin{aligned}
S'_{min} = &\frac{1}{2\,g^2}\int d^4x\ d^4\theta\ \Big[\ \mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\ \overline{W}_{\dot\alpha}\right)\\
&\qquad\qquad +\mathcal{F}^2\ \overline{\theta}^2\ \mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\overline{W}_{\dot\alpha}\right)\mathrm{Tr}\left(\overline{W}^{\dot\beta}\overline{W}_{\dot\beta}\right)\Big]\\
&+\frac{1}{2\,g_0^2\,\mathcal{N}}\int d^4x\ d^4\theta\ \Big[\ \mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\right)\mathrm{Tr}\left(\overline{W}_{\dot\alpha}\right)\\
&\qquad\qquad +4i\mathcal{F}^{\rho\gamma}\ \overline{\theta}^2\ \mathrm{Tr}\left(\partial_{\rho\dot\rho}\overline{\Gamma}^{\dot\alpha}\right)\mathrm{Tr}\left(\overline{W}_{\dot\alpha}\overline{\Gamma}_{\gamma}{}^{\dot\rho}\right)\Big]\\
&+\frac{1}{l^2}\,\mathcal{F}^2\int d^4x\ d^4\theta\ \overline{\theta}^2\ \mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\ \overline{W}_{\dot\alpha}\ \overline{W}^{\dot\beta}\ \overline{W}_{\dot\beta}\right)
\end{aligned}\tag{1.5}$$

depending on just three arbitrary coupling constants, of which two, the $SU(\mathcal{N})$ and the $U(1)$ g and g_0, reflect the corresponding situation in ordinary AC SYM, whereas the third one is a new feature of NAC.

Final comments for the exhausted reader

Let us summarize this summary:

We have examined in superspace the quantum properties of NAC SYM, concluding that, as in components, suitable deformations of the classical actions are necessary in

order to achieve renormalizability. This is only at the one–loop level; the higher–loop situation remains to be studied but the hope is that, just as in the Wess-Zumino model [10, 18], these deformations are sufficient to achieve the same results at all loops. We note that the deformed actions can also be written purely in terms of star products and that the deformation parameter needs no renormalization.

The reader may feel that the corresponding analysis in components is simpler, but it does require going to WZ gauge with implications that were not *a priori* clear to us. We also think that some of the other issues that arose in the course of our investigation are of interest, if only to superspace *aficionados*.

Our work complements, and should be compared to that of [12]. In fact, by taking our superspace results to WZ component gauge it is possible to make a direct comparison. In general we are in agreement with the results there although some of the details may differ and are also obscured by the choice of WZ gauge and elimination of auxiliary fields. Working in superspace, or with the full complement of auxiliary fields, automatically obviates the need for a nonlinear, field–dependent wave-function renormalization (see also [12, 27]) and makes the discussion cleaner. Aside from this, the only significant difference seems to be the following: the new term in the modified component action of that reference that is required for renormalization (besides the splitting between g and g_0 terms) would come from a superspace expression of the form $\int \overline{\theta}^2 \mathrm{Tr}(\overline{\Gamma})\mathrm{Tr}(\overline{W})\mathrm{Tr}(\overline{W}\overline{W})$. Instead, our corresponding new term has the form $\int \overline{\theta}^2 \mathrm{Tr}(\overline{\Gamma}\overline{W}\overline{W}\overline{W})$. The two terms differ only in the color structure. It would be nice to understand the reason for this mismatch.

We remarked earlier that starting with the original deformed action the complete contribution to the one-loop divergences comes only from chiral matter or ghost fields. It follows immediately that at least the gauge sector is completely finite in a theory with the field content of $N = 4$ theory consisting of one gauge and three matter chiral fields (see 6.4). The situation for the final modified actions is more complicated. In such a theory one would expect that corresponding modifications in the chiral matter sector would lead to additional one–loop contributions to the gauge effective action. We conjecture that under those circumstances the modified actions may very well maintain the finite properties of the undeformed $N = 4$ theory.

In ref.[5] the classical action for NAC SYM in components, in the WZ gauge has been derived directly from string theory by computing the low energy limit of string scattering amplitudes in the presence of a RR two–form flux. The resulting action has the form (1.1) and it corresponds to the natural NAC generalization of the ordinary action where products have been promoted to star products. It would be interesting to investigate how to extend that analysis to reproduce the renormalizable actions (1.4, 1.5) with the correct number of coupling constants directly from string theory.

2 SYM theories in $N = 1/2$ superspace

Nonanticommutative $N = (\frac{1}{2}, 0)$ superspace can be defined as a truncation of euclidean $N = (1, 1)$ superspace endowed with nonstandard hermitian conjugation rules for the spinorial variables [2, 17, 28]. It is described by the set of coordinates $(x^{\alpha\dot{\alpha}}, \theta^\alpha, \overline{\theta}^{\dot{\alpha}})$, $(\theta^\alpha)^* = i\theta_\alpha$, $(\overline{\theta}^{\dot{\alpha}})^* = i\overline{\theta}_{\dot{\alpha}}$, satisfying

$$\{\theta^\alpha, \theta^\beta\} = 2\mathcal{F}^{\alpha\beta} \qquad \text{the rest} = 0 \tag{2.1}$$

where $\mathcal{F}^{\alpha\beta}$ is a 2×2 symmetric, constant matrix. This algebra is consistent only in euclidean signature where the chiral and antichiral sectors are totally independent and not related by complex conjugation.

We use chiral representation [26] for supercharges and covariant derivatives

$$\begin{aligned} \overline{Q}_{\dot{\alpha}} &= i(\overline{\partial}_{\dot{\alpha}} - i\theta^\alpha \partial_{\alpha\dot{\alpha}}) \quad , \quad Q_\alpha = i\partial_\alpha \\ \overline{D}_{\dot{\alpha}} &= \overline{\partial}_{\dot{\alpha}} \quad , \quad D_\alpha = \partial_\alpha + i\overline{\theta}^{\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \end{aligned} \tag{2.2}$$

While the algebra of covariant derivatives is not modified, the algebra of supercharges receives an extra contribution from (2.1) and the supersymmetry in the antichiral sector is explicitly broken [2].

We note that in euclidean signature the complex conjugation relations are

$$\begin{aligned} Q^*_\alpha &= iQ^\alpha \quad , \quad \overline{Q}^*_{\dot{\alpha}} = i\overline{Q}^{\dot{\alpha}} \\ D^*_\alpha &= -iD^\alpha \quad , \quad \overline{D}^*_{\dot{\alpha}} = -i\overline{D}^{\dot{\alpha}} \end{aligned} \tag{2.3}$$

On the class of smooth superfunctions $\phi(x^{\alpha\dot{\alpha}}, \theta^\alpha, \overline{\theta}^{\dot{\alpha}})$, the NAC geometry (2.1) can be realized by introducing the nonanticommutative but associative star product

$$\begin{aligned} \phi * \psi &\equiv \phi e^{-\overleftarrow{\partial}_\alpha \mathcal{F}^{\alpha\beta} \overrightarrow{\partial}_\beta} \psi \\ &= \phi\psi - \phi \overleftarrow{\partial}_\alpha \mathcal{F}^{\alpha\beta} \overrightarrow{\partial}_\beta \psi + \frac{1}{2} \phi \overleftarrow{\partial}_\alpha \overleftarrow{\partial}_\gamma \mathcal{F}^{\alpha\beta} \mathcal{F}^{\gamma\delta} \overrightarrow{\partial}_\delta \overrightarrow{\partial}_\beta \psi \\ &= \phi\psi - \phi \overleftarrow{\partial}_\alpha \mathcal{F}^{\alpha\beta} \overrightarrow{\partial}_\beta \psi - \frac{1}{2} \mathcal{F}^2 \partial^2 \phi \, \partial^2 \psi \end{aligned} \tag{2.4}$$

where we have defined $\mathcal{F}^2 \equiv \mathcal{F}^{\alpha\beta}\mathcal{F}_{\alpha\beta}$. The covariant derivatives (2.2) are still derivations for this product. Therefore, the class of (anti)chiral superfields defined by the constraints $\overline{D}_{\dot{\alpha}} * \Phi = D_\alpha * \overline{\Phi} = 0$ is closed.

A property of the star product that we will often use is the following: The trace of the $*$–product of a number of fields is not in general cyclic unless it is integrated over $d^2\theta$. Specifically, we have

$$\mathrm{Tr}(A * B) \neq \mathrm{Tr}(B * A) \tag{2.5}$$

$$\int d^2\overline{\theta} \; \mathrm{Tr}(A * B) \neq \int d^2\overline{\theta} \; \mathrm{Tr}(B * A) \tag{2.6}$$

but

$$\int d^2\theta \ \mathrm{Tr}(A * B) = \int d^2\theta \ \mathrm{Tr}(B * A) \tag{2.7}$$

However, even under $d^2\theta$ integration the cyclicity property gets spoiled when the trace appears multiplied by an extra function. In particular,

$$\int d^2\theta \ \mathrm{Tr}(A * B)\mathrm{Tr}(C) \neq \int d^2\theta \ \mathrm{Tr}(B * A)\mathrm{Tr}(C) \tag{2.8}$$

These properties can be easily proved by expanding the star product as in (2.4).

We now turn to the description of SYM theories in the non(anti)commutative case. Supersymmetric Yang–Mills theories in $N = 1/2$ superspace can be defined as usual in terms of a scalar prepotential V in the adjoint representation of the gauge group ($V \equiv V_A T^A$, T^A being the group generators) [§]. It is subject to the supergauge transformations

$$e_*^V \rightarrow e_*^{V'} = e_*^{i\overline{\Lambda}} * e_*^V * e_*^{-i\Lambda} \tag{2.9}$$

where $\Lambda, \overline{\Lambda}$ are chiral and antichiral superfields, respectively.

Supergauge covariant derivatives in superspace can be defined in the so–called *gauge chiral representation* as

$$\nabla_A \equiv (\nabla_\alpha, \nabla_{\dot\alpha}, \nabla_{\alpha\dot\alpha}) \ = \ (e_*^{-V} * D_\alpha \, e_*^V \ , \ \overline{D}_{\dot\alpha} \ , \ \ -i\{\nabla_\alpha, \nabla_{\dot\alpha}\}_*) \tag{2.10}$$

or, equivalently, in *gauge antichiral representation* as

$$\overline{\nabla}_A \equiv (\overline{\nabla}_\alpha, \overline{\nabla}_{\dot\alpha}, \overline{\nabla}_{\alpha\dot\alpha}) \ = \ (D_\alpha \ , \ e_*^V * \overline{D}_{\dot\alpha} \, e_*^{-V} \ , \ \ -i\{\overline{\nabla}_\alpha, \overline{\nabla}_{\dot\alpha}\}_*) \tag{2.11}$$

In contradistinction to the ordinary anticommutative case where the superfield V is chosen to be real, in the NAC case, in order to preserve the c.c. relations (2.3) also for the supergauge covariant derivatives, we choose V to be *pure imaginary*, $V^\dagger = -V$.

The supergauge covariant derivatives can be expressed in terms of ordinary superspace derivatives and a set of connections, as $\nabla_A \equiv D_A - i\Gamma_A$ or $\overline{\nabla}_A \equiv \overline{D}_A - i\overline{\Gamma}_A$. In chiral representation they are

$$\Gamma_\alpha = ie_*^{-V} * D_\alpha \, e_*^V \qquad , \qquad \Gamma_{\dot\alpha} = 0 \qquad , \qquad \Gamma_{\alpha\dot\alpha} = -i\overline{D}_{\dot\alpha}\Gamma_\alpha \tag{2.12}$$

whereas in antichiral representation we have

$$\overline{\Gamma}_\alpha = 0 \qquad , \qquad \overline{\Gamma}_{\dot\alpha} = ie_*^V * \overline{D}_{\dot\alpha} \, e_*^{-V} \qquad , \qquad \overline{\Gamma}_{\alpha\dot\alpha} = -iD_\alpha\overline{\Gamma}_{\dot\alpha} \tag{2.13}$$

[§]Since in the presence of non(anti)commutativity also the $U_*(1)$ gauge theory becomes nonabelian the relations we introduce hold nontrivially for any gauge group, $U_*(1)$ included.

The field strengths are defined as $*$–commutators of supergauge covariant derivatives

$$\text{chiral repr.} \qquad W_\alpha = -\frac{1}{2}[\nabla^{\dot\alpha}, \nabla_{\alpha\dot\alpha}]_* \qquad , \qquad \widetilde{W}_{\dot\alpha} = -\frac{1}{2}[\nabla^{\alpha}, \nabla_{\alpha\dot\alpha}]_* \tag{2.14}$$

$$\text{antich. repr.} \quad \widetilde{W}_\alpha = -\frac{1}{2}[\overline{\nabla}^{\dot\alpha}, \overline{\nabla}_{\alpha\dot\alpha}]_* \qquad , \qquad \overline{W}_{\dot\alpha} = -\frac{1}{2}[\overline{\nabla}^{\alpha}, \overline{\nabla}_{\alpha\dot\alpha}]_* \tag{2.15}$$

and satisfy the Bianchi's identities $\nabla^\alpha * W_\alpha + \nabla^{\dot\alpha} * \widetilde{W}_{\dot\alpha} = 0$ and $\overline{\nabla}^\alpha * \widetilde{W}_\alpha + \overline{\nabla}^{\dot\alpha} * \overline{W}_{\dot\alpha} = 0$. The superfield strengths in antichiral representation are related to the ones in chiral representation as

$$\widetilde{W}_\alpha = e_*^V * W_\alpha * e_*^{-V} \qquad , \qquad \overline{W}_{\dot\alpha} = e_*^V * \widetilde{W}_{\dot\alpha} * e_*^{-V} \tag{2.16}$$

While W_α and $\overline{W}_{\dot\alpha}$ are ordinary chiral and antichiral superfields, the tilde quantities are covariantly (anti)chiral.

Under supergauge transformations (2.9) all the superfield strengths transform covariantly. For infinitesimal transformations we have

$$\begin{aligned} \delta W_\alpha = i[\Lambda, W_\alpha]_* \qquad &, \qquad \delta\widetilde{W}_{\dot\alpha} = i[\Lambda, \widetilde{W}_{\dot\alpha}]_* \\ \delta\widetilde{W}_\alpha = i[\overline{\Lambda}, \widetilde{W}_\alpha]_* \qquad &, \qquad \delta\overline{W}_{\dot\alpha} = i[\overline{\Lambda}, \overline{W}_{\dot\alpha}]_* \end{aligned} \tag{2.17}$$

If we expand $W_\alpha = W_\alpha^A T^A$ where T^A are the group generators, use the definitions (A.3, A.4) and the identity (A.7) we can rewrite

$$\delta W_\alpha^A = \frac{i}{2} d_{ABC}[\Lambda^B, W_\alpha^C]_* - \frac{1}{2} f_{ABC}\{\Lambda^B, W_\alpha^C\}_* \tag{2.18}$$

and similarly for the others. In the particular case of $U(\mathcal{N})$, given the explicit expressions (A.4) for d_{ABC}, the first term in δW_α^A mixes $U(1)$ and $SU(\mathcal{N})$ fields. In particular, the abelian, $U(1)$ field strength W_α^0 transforms nontrivially under $SU(\mathcal{N})$ and its transform is given in terms of both $U(1)$ and $SU(\mathcal{N})$ fields. In the commutative limit this term goes to zero and we are back to the ordinary theory where $SU(\mathcal{N})$ fields only transform under $SU(\mathcal{N})$ transformations while the abelian field is a singlet. As we shall see this is the source of significant complications.

In the ordinary anticommutative superspace, in the absence of instantonic effects, any of the following actions

$$\begin{aligned} S = \int d^4x d^2\theta \ \mathrm{Tr}(W^\alpha W_\alpha) \quad &; \quad \widetilde{S} = \int d^4x d^2\overline{\theta} \ \mathrm{Tr}(\widetilde{W}^{\dot\alpha}\widetilde{W}_{\dot\alpha}) \\ \overline{S} = \int d^4x d^2\overline{\theta} \ \mathrm{Tr}(\overline{W}^{\dot\alpha}\overline{W}_{\dot\alpha}) \quad &; \quad \widetilde{\overline{S}} = \int d^4x d^2\theta \ \mathrm{Tr}(\widetilde{W}^{\alpha}\widetilde{W}_{\alpha}) \end{aligned} \tag{2.19}$$

can be used to describe pure gauge theory. In fact, any of these actions is gauge invariant and, when reduced to components in the WZ gauge, describes the correct dynamics of

the physical degrees of freedom (gluons and gluinos) [26]. In particular, the actions S and $\widetilde{\widetilde{S}}$, as well as $\overline{S}$ and $\widetilde{S}$, are trivially identical as can be easily understood by using the relations (2.16) and the cyclicity of the trace. Instead, S and $\overline{S}$ differ by surface terms which are zero if we do not include instantons. The equivalence of the actions (2.19) holds for any gauge group, $U(1)$ included.

A peculiarity of the NAC case is that in the presence of star products it is no longer true that the four actions (2.19) are all equivalent. For example, let us consider $\widetilde{S}$ versus $\overline{S}$. By using the relations (2.16) we have the following chain of relations

$$\begin{aligned} \widetilde{S} = \int d^4x d^2\overline{\theta}\ \mathrm{Tr}(\widetilde{W}^{\dot{\alpha}}\widetilde{W}_{\dot{\alpha}}) &= \int d^4x d^2\overline{\theta}\ \mathrm{Tr}(e_*^{-V} * \overline{W}^{\dot{\alpha}}\overline{W}_{\dot{\alpha}} * e_*^{V}) \\ &\neq \int d^4x d^2\overline{\theta}\ \mathrm{Tr}(\overline{W}^{\dot{\alpha}}\overline{W}_{\dot{\alpha}}) = \overline{S} \end{aligned} \tag{2.20}$$

since in this case the trace is not cyclic, as follows from (2.5). What is interesting from the physical point of view is that the non–equivalence of the two actions has important consequences for their gauge invariance. In fact, it is easy to show that under transformations (2.17) the action $\overline{S}$ *is* gauge invariant whereas $\widetilde{S}$ *is not*. For what concerns S and $\widetilde{\widetilde{S}}$ instead, they are still equivalent and both gauge invariant since they are defined as chiral integrals and the cyclicity of the trace can be used in this case. Finally, as in the ordinary case, the two gauge invariant actions S and $\overline{S}$ are equivalent up to instantonic effects when reduced to components in the WZ gauge [2].

The situation is even worse if we consider only the $U(1)$ part of the actions (2.19). We note that this part can be separated out in the form of a product of single traces. Looking at the $U(\mathcal{N})$ transformations of the abelian superfield strengths as given in (2.18) one can prove that among the abelian actions

$$\begin{aligned} S_0 &= \int d^4x d^2\theta\ \mathrm{Tr}(W^\alpha)\mathrm{Tr}(W_\alpha) \quad ; \quad \widetilde{S}_0 = \int d^4x d^2\overline{\theta}\ \mathrm{Tr}(\widetilde{W}^{\dot{\alpha}})\mathrm{Tr}(\widetilde{W}_{\dot{\alpha}}) \\ \overline{S}_0 &= \int d^4x d^2\overline{\theta}\ \mathrm{Tr}(\overline{W}^{\dot{\alpha}})\mathrm{Tr}(\overline{W}_{\dot{\alpha}}) \quad ; \quad \widetilde{\widetilde{S}}_0 = \int d^4x d^2\theta\ \mathrm{Tr}(\widetilde{W}^{\alpha})\mathrm{Tr}(\widetilde{W}_{\alpha}) \end{aligned} \tag{2.21}$$

only $\widetilde{\widetilde{S}}_0$ is gauge invariant, whereas the others are *not* and need to be completed by extra terms in order to restore gauge invariance. In particular, we will be interested in the gauge invariant completion of $\overline{S}_0$ which reads [21]

$$\int d^4x\ d^2\overline{\theta}\ \mathrm{Tr}(\overline{W}^{\dot{\alpha}})\mathrm{Tr}(\overline{W}_{\dot{\alpha}}) + 4i\mathcal{F}^{\rho\gamma} \int d^4x\ d^4\theta\ \ \overline{\theta}^2\ \mathrm{Tr}\left(\partial_{\rho\dot{\rho}}\overline{\Gamma}^{\dot{\alpha}}\right)\mathrm{Tr}\left(\overline{W}_{\dot{\alpha}}\overline{\Gamma}_{\gamma}^{\ \dot{\rho}}\right) \tag{2.22}$$

where $\mathcal{F}^{\rho\gamma}$ is the NAC parameter. We note that the lack of invariance of the abelian actions in (2.21) is due to the fact that the abelian gauge field transforms nontrivially under $SU(\mathcal{N})$ and its variation is proportional to the $SU(\mathcal{N})$ gauge fields (see eq. (2.18)). We also note that despite the nontrivial variation of the $U(1)$ part as described in $\overline{S}_0$,

the total action $\overline{S}$ which describes the propagation of $U(1)$ and $SU(\mathcal{N})$ fields *is* gauge invariant since the gauge variation of the $U(1)$ fields gets compensated by the gauge variation of the $SU(\mathcal{N})$ fields. This is peculiar to the NAC case and does not have direct correspondence in the ordinary anticommutative case.

Given the asymmetry between chiral and antichiral representations introduced by the nonanticommutativity, it turns out that the choice of one representation with respect to the other may be preferable from the point of view of technical convenience. We find it preferable to work in antichiral representation for the following reason: in the ordinary, anticommuting case we often switch between full superspace integrals and chiral (or antichiral) integrals by using the equivalence $\int d^4xd^4\theta \ \mathrm{Tr}[F(z)] \equiv \int d^4xd^2\theta \ \mathrm{Tr}[\overline{\nabla}^2 F(z)] \equiv \int d^4xd^2\overline{\theta} \ \mathrm{Tr}[\nabla^2 F(z)]$. However, in the NAC case the second equality fails if one is working in chiral representation, as can be seen by examining its derivation in the following sequence of equalities (star products understood in the NAC case):

$$\int d^4xd^4\theta \ \mathrm{Tr}[F(z)] = \int d^4xd^4\theta \ \mathrm{Tr}\{e^{-V}[F(z)]e^V\} = \int d^4xd^2\overline{\theta} \ \mathrm{Tr}\{D^2 e^{-V}[F(z)]e^V\}$$
$$= \int d^4xd^2\overline{\theta} \ \mathrm{Tr}\{e^{-V}e^V D^2 e^{-V}[F(z)]e^V\} = \int d^4xd^2\overline{\theta} \ \mathrm{Tr}\{e^{-V}\nabla^2[F(z)]e^V\} \qquad (2.23)$$

(Note that $\nabla^2 e^V = 0$.) In the ordinary case one can use the cyclicity of the trace to remove the exponentials after the last equality and thus establish the required equivalence. However, in the NAC case we know that the cyclicity of the trace does not hold since a $d^2\theta$ integration is lacking. Thus the first step above, which introduces the exponentials, is valid; however, after the last step the exponentials cannot be removed and the usual equivalence fails. By working in antichiral representation we generally manage to avoid this problem since $\nabla_\alpha = D_\alpha$. We note that the same problem does not occur for $\overline{\nabla}_{\dot{\alpha}}$ since the surviving $d^2\theta$ integration makes the trace cyclic as in the ordinary case.

Therefore, from now on we choose to describe the gauge sector of the theory in antichiral representation with the classical action

$$S_{inv} = \frac{1}{2g^2} \int d^4xd^2\overline{\theta} \ \mathrm{Tr}(\overline{W}^{\dot{\alpha}}\overline{W}_{\dot{\alpha}}) \qquad (2.24)$$

or more generally with

$$S_{inv} = \frac{1}{2g^2} \int d^4xd^2\overline{\theta} \ \mathrm{Tr}(\overline{W}^{\dot{\alpha}}\overline{W}_{\dot{\alpha}}) \qquad (2.25)$$
$$+ \frac{1}{2g_0^2} \int d^4x \ d^4\theta \ \Big[\ \mathrm{Tr}(\overline{\Gamma}^{\dot{\alpha}})\mathrm{Tr}(\overline{W}_{\dot{\alpha}}) + 4i\mathcal{F}^{\rho\gamma}\overline{\theta}^2 \ \mathrm{Tr}\left(\partial_{\rho\dot{\rho}}\overline{\Gamma}^{\dot{\alpha}}\right) \mathrm{Tr}\left(\overline{W}_{\dot{\alpha}}\overline{\Gamma}_{\gamma}{}^{\dot{\rho}}\right) \Big]$$

if we are interested in assigning different coupling constants to the $SU(\mathcal{N})$ and $U(1)$ gauge fields. An equivalently convenient choice would make use of $\widetilde{\overline{S}}$ in (2.19) and $\widetilde{\overline{S}}_0$ in (2.21).

3 Background field method in $N = 1/2$ superspace

In the case of ordinary (anti)commutative superspace a suitable procedure for performing perturbative quantum calculations for super Yang–Mills theories is the background field method [29, 26]. This method can be applied to pure gauge theories or to gauge theories in the presence of matter in arbitrary representations of the gauge group (although at one loop, for matter in complex representations, the "doubling trick" [26] cannot be used). It consists of a nonlinear quantum–background splitting on the gauge superfields (reflecting the nonlinear nature of the gauge transformations) which leads to separate background and quantum gauge invariances. Gauge fixing is then chosen which breaks the quantum invariance while keeping manifest invariance with respect to the background gauge transformations. Therefore, at any given order in the loop expansion the contributions to the effective action are expressed directly in terms of covariant derivatives and field strengths (without explicit dependence on the prepotential V).

We generalize the background field method to the case of NAC super Yang–Mills theories with chiral matter in a *real* representation of the gauge group. We perform the splitting by writing the covariant derivatives ∇_A in terms of *background* covariant derivatives $\boldsymbol{\nabla}_A$ and, in contradistinction to the usual Lorentzian case, the pure imaginary *quantum* prepotential V. In the ordinary case this splitting is usually done either in *chiral representation* (the customary case) by writing $\nabla_\alpha = e^{-V}\boldsymbol{\nabla}_\alpha e^V$, $\nabla_{\dot\alpha} = \boldsymbol{\nabla}_{\dot\alpha}$ or in *antichiral representation* where $\nabla_\alpha = \boldsymbol{\nabla}_\alpha$, $\nabla_{\dot\alpha} = e^V\boldsymbol{\nabla}_{\dot\alpha}e^{-V}$. At the same time covariantly (anti)chiral superfields (e.g. in the adjoint representation – the case we consider here) are expressed in terms of background covariantly (anti)chiral objects as $\Phi = \boldsymbol{\Phi}$, $\overline{\Phi} = e^{-V}\overline{\boldsymbol{\Phi}}e^V$ or $\Phi = e^V\boldsymbol{\Phi}e^{-V}$, $\overline{\Phi} = \overline{\boldsymbol{\Phi}}$, respectively. The background covariant derivatives are assumed to be in *vector* representation.

In the NAC case, as discussed in the previous Section, it turns out to be more convenient to work in quantum antichiral representation. It also turns out to be possible and more convenient to choose the background derivatives in antichiral representation. Thus, in the NAC case we use the splitting

$$\begin{aligned}
&\nabla_\alpha = \boldsymbol{\nabla}_\alpha = D_\alpha \quad , \quad \nabla_{\dot\alpha} = e_*^V * \boldsymbol{\nabla}_{\dot\alpha} * e_*^{-V} = e_*^V * e_*^U * \overline{D}_{\dot\alpha}\, e_*^{-U} * e_*^{-V}\\
&\overline{\Phi} = \overline{\boldsymbol{\Phi}} \quad , \quad \Phi = e_*^V * \boldsymbol{\Phi} * e_*^{-V} = e_*^V * e_*^U * \phi * e_*^{-U} * e_*^{-V}
\end{aligned} \tag{3.1}$$

The splitting of the Euclidean prepotential $e_*^V \to e_*^V * e_*^U$ where U is the background prepotential, is different from the Lorentzian case where the reality of V forces us to choose a more complicated one [26] and precludes the possibility of having all three types of derivatives in (anti)chiral representation.

The derivatives transform covariantly with respect to two types of gauge transformations: quantum transformations

$$\begin{aligned}
&e_*^V \to e_*^{i\overline{\Lambda}} * e_*^V * e_*^{-i\Lambda} \quad , \quad e_*^U \to e_*^U\\
&\nabla_A \to e_*^{i\overline{\Lambda}} * \nabla_A * e_*^{-i\overline{\Lambda}} \quad , \quad \boldsymbol{\nabla}_A \to \boldsymbol{\nabla}_A
\end{aligned} \tag{3.2}$$

with background covariantly (anti)chiral parameters, $\boldsymbol{\nabla}_\alpha \overline{\Lambda} = \boldsymbol{\nabla}_{\dot\alpha} \Lambda = 0$, and background transformations

$$
\begin{array}{lcl}
e_*^V \to e_*^{i\overline{\lambda}} * e_*^V * e_*^{-i\overline{\lambda}} & , & e_*^U \to e_*^{i\overline{\lambda}} * e_*^U * e_*^{-i\lambda} \\
\nabla_A \to e_*^{i\overline{\lambda}} * \nabla_A * e_*^{-i\overline{\lambda}} & , & \boldsymbol{\nabla}_A \to e_*^{i\overline{\lambda}} * \boldsymbol{\nabla}_A * e_*^{-i\overline{\lambda}}
\end{array}
\tag{3.3}
$$

with ordinary chiral parameters $\overline{D}_{\dot\alpha} \lambda = D_\alpha \overline{\lambda} = 0$

Under quantum transformations background covariantly (anti)chiral fields (in the adjoint representation) transform as $\boldsymbol{\Phi}' = e_*^{i\Lambda} * \boldsymbol{\Phi} * e_*^{-i\Lambda}$, $\overline{\boldsymbol{\Phi}}' = e_*^{i\overline{\Lambda}} * \overline{\boldsymbol{\Phi}} * e_*^{-i\overline{\Lambda}}$; under background transformations they both transform covariantly with parameter $\overline{\lambda}$, $\boldsymbol{\Phi}' = e_*^{i\overline{\lambda}} * \boldsymbol{\Phi} * e_*^{-i\overline{\lambda}}$, $\overline{\boldsymbol{\Phi}}' = e_*^{i\overline{\lambda}} * \overline{\boldsymbol{\Phi}} * e_*^{-i\overline{\lambda}}$. Under both quantum and background transformations the *full* (anti)chiral fields transform covariantly with the parameters $\overline{\Lambda}$ and $\overline{\lambda}$ respectively.

The classical action (2.24) for a pure gauge theory, or more generally the action

$$
\begin{aligned}
S &= \frac{1}{2g^2} \int d^4x \; d^2\overline{\theta} \; \mathrm{Tr}(\overline{W}^{\dot\alpha} \overline{W}_{\dot\alpha}) + \int d^4x \; d^4\theta \; \mathrm{Tr}(e_*^{-V} * \overline{\Phi} * e_*^V * \Phi) \\
&\quad - \frac{1}{2} m \int d^4x \; d^2\theta \; \mathrm{Tr}(\Phi^2) - \frac{1}{2} \overline{m} \int d^4x \; d^2\overline{\theta} \; \mathrm{Tr}(\overline{\Phi}^2)
\end{aligned}
\tag{3.4}
$$

for gauge plus covariantly chiral matter is invariant under the transformations (3.2, 3.3). Background field quantization consists in performing gauge–fixing which explicitly breaks the (3.2) gauge invariance while preserving manifest invariance of the effective action and correlation functions under (3.3). Choosing as in the ordinary case the gauge–fixing functions as $f = \overline{\boldsymbol{\nabla}}^2 * V$, $\overline{f} = \boldsymbol{\nabla}^2 * V$ the resulting gauge–fixed action has exactly the same structure as in the ordinary case [29, 26] with products promoted to star products. Precisely, $S_{tot} = S_{inv} + S_{GF} + S_{gh}$ where S_{gh} is given in terms of background covariantly (anti)chiral FP and NK ghost superfields as

$$
S_{gh} = \int d^4x d^4\theta \left[\overline{c}' c - c' \overline{c} + ... + \overline{b} b \right]
\tag{3.5}
$$

We now concentrate on the pure gauge part of the action and derive the Feynman rules suitable for one–loop calculations, starting from

$$
S_{inv} + S_{GF} = -\frac{1}{2g^2} \int d^4x d^4\theta \; \mathrm{Tr}[(e_*^V * \overline{\boldsymbol{\nabla}}^{\dot\alpha} * e_*^{-V}) * D^2 (e_*^V * \overline{\boldsymbol{\nabla}}_{\dot\alpha} * e_*^{-V}) + \frac{1}{\alpha} V * (\overline{\boldsymbol{\nabla}}^2 D^2 + D^2 \overline{\boldsymbol{\nabla}}^2) * V]
\tag{3.6}
$$

Having chosen antichiral representation, we were able to replace one factor of $\boldsymbol{\nabla}^2 \equiv D^2$ by $\int d^2\theta$. As explained in the previous Section, in the chiral representation the corresponding replacement of $\boldsymbol{\nabla}$ would have been fraught with difficulties in the NAC case.

We extract the quadratic part in V and read the V-V propagator and the V-V-background vertices which are the only vertices entering one–loop diagrams. We proceed

as in the ordinary case, replacing products with star products. The propagator is the ordinary one since the star product does not affect quadratic actions. Working in Feynman gauge ($\alpha = 1$) it is

$$\langle V^A(z) V^B(z') \rangle = \frac{\delta^{AB}}{\Box_0} \, \delta^{(8)}(z - z') \tag{3.7}$$

where $\Box_0 \equiv \frac{1}{2}\partial^{\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}}$. For the interaction terms, after a bit of algebra we find

$$-\frac{1}{2g^2}\int d^4x d^4\theta \; \mathrm{Tr}\Big\{ V * \Big[-i[\mathbf{\Gamma}^a, \partial_a V]_* - i\{\widetilde{\mathbf{W}}^\alpha, D_\alpha V\}_* - i\{\overline{\mathbf{W}}^{\dot{\alpha}}, \overline{D}_{\dot{\alpha}} V\}_* \\ - \frac{i}{2}[\partial_a \mathbf{\Gamma}^a, V]_* - \frac{1}{2}[\mathbf{\Gamma}^a, [\mathbf{\Gamma}_a, V]]_* - \{\overline{\mathbf{W}}^{\dot{\alpha}}, [\overline{\mathbf{\Gamma}}_{\dot{\alpha}}, V]_*\}_* \Big] \Big\} \tag{3.8}$$

where $\mathbf{\Gamma}, \mathbf{W}$ are background quantities.

We now turn to the action for full (not background) covariantly chiral matter and review first the usual, anticommutative case, with

$$\begin{aligned} S = \quad & \int d^4x d^4\theta \; \overline{\Phi}\Phi = \int d^4x d^4\theta \; \overline{\mathbf{\Phi}} e^V \mathbf{\Phi} e^{-V} \\ & = \int d^4x d^4\theta \; \overline{\mathbf{\Phi}}\mathbf{\Phi} + \overline{\mathbf{\Phi}}[V, \mathbf{\Phi}] + \cdots \end{aligned} \tag{3.9}$$

Here Φ and $\overline{\Phi}$ are related by complex conjugation. The first term in the expansion is the kinetic term for background covariantly (anti)chiral fields. In particular, ghosts fall in this category so the following procedure can be applied to the action (3.5). The remaining terms give rise to ordinary interactions with the quantum field V and can be treated in standard perturbative fashion. Here, for one-loop calculations, we concentrate on the kinetic term. The corresponding equations of motion

$$\mathcal{O}\begin{pmatrix} \mathbf{\Phi} \\ \overline{\mathbf{\Phi}} \end{pmatrix} = 0 \qquad \mathcal{O} = \begin{pmatrix} 0 & \overline{\mathbf{\nabla}}^2 \\ D^2 & 0 \end{pmatrix} \tag{3.10}$$

can be formally derived from the functional determinant

$$\Delta \; = \; \int \mathcal{D}\Psi e^{\overline{\Psi}\mathcal{O}\Psi} \sim (\det\mathcal{O})^{-\frac{1}{2}} \tag{3.11}$$

where Ψ is the column vector $\begin{pmatrix} \mathbf{\Phi} \\ \overline{\mathbf{\Phi}} \end{pmatrix}$. If we perform the change of variables $\Psi = \sqrt{\mathcal{O}}\Psi'$, with jacobian $\det\sqrt{\mathcal{O}} = \Delta^{-\frac{1}{2}}$, we can write

$$\Delta = \int \mathcal{D}\Psi' \; \Delta^{-1} \; e^{\overline{\Psi}'\mathcal{O}^2\Psi'} \tag{3.12}$$

or equivalently

$$\Delta^2 = \int \mathcal{D}\Psi e^{\overline{\Psi}\mathcal{O}^2\Psi} \tag{3.13}$$

where

$$\mathcal{O}^2 = \begin{pmatrix} \overline{\nabla}^2 D^2 & 0 \\ 0 & D^2 \overline{\nabla}^2 \end{pmatrix} \tag{3.14}$$

is a diagonal matrix. The corresponding equations of motion can be derived from the (anti)chiral actions

$$\begin{aligned} \overline{S}' &= \frac{1}{2} \int d^4x d^4\theta \; \overline{\mathbf{\Phi}} \; \overline{\nabla}^2 \; \overline{\mathbf{\Phi}} \; = \; \frac{1}{2} \int d^4x d^2\overline{\theta} \; \overline{\mathbf{\Phi}} \; \Box_- \; \overline{\mathbf{\Phi}} \\ S' &= \frac{1}{2} \int d^4x d^4\theta \; \mathbf{\Phi} \; D^2 \; \mathbf{\Phi} \; = \; \frac{1}{2} \int d^4x d^2\theta \; \mathbf{\Phi} \; \Box_+ \; \mathbf{\Phi} \end{aligned} \tag{3.15}$$

with $\Box_\pm$ defined in refs. ([29, 26]). Note that in the NAC case the second expression would not hold in chiral representation.

In the ordinary case the following chain of identities holds [26]

$$\Delta^2 = \int \mathcal{D}\mathbf{\Phi}\mathcal{D}\overline{\mathbf{\Phi}} \; e^{S'+\overline{S}'} \; = \; \Big| \int \mathcal{D}\mathbf{\Phi} e^{S'} \Big|^2 \; = \; \Big(\int \mathcal{D}\mathbf{\Phi} e^{S'} \Big)^2 \tag{3.16}$$

where we have used $\overline{S}' = (S')^\dagger$ and the fact that they both contribute in the same way to Δ [26]. Therefore, when Δ is real, we can identify the original Δ with $\int \mathcal{D}\mathbf{\Phi} e^{S'}$ and derive from here the Feynman rules [26].

We now extend the previous derivation to the case of NAC euclidean superspace where all the h.c. relations are relaxed and $\mathbf{\Phi}$, $\overline{\mathbf{\Phi}}$ are two independent *but real* superfields. The matter action is still given by (3.9) and we can still define Δ as in (3.12). Therefore, we write

$$\Delta_* = \int \mathcal{D}\Psi \; e^{\Psi^T * \mathbf{O} * \Psi} \sim (\det(\mathbf{O}))^{-\frac{1}{2}} \tag{3.17}$$

We can then proceed as before and square the functional integral to obtain

$$\Delta_*^2 = \int \mathcal{D}\Psi^T e^{\Psi * \mathbf{O}^2 * \Psi} \tag{3.18}$$

where $\mathbf{O}^2$ is given in (3.14) with the products promoted to star products. Now, if we introduce

$$\Delta_1 = \int \mathcal{D}\mathbf{\Phi} e^{S'} \qquad , \qquad \Delta_2 = \int \mathcal{D}\overline{\mathbf{\Phi}} e^{\overline{S}'} \tag{3.19}$$

with S', $\overline{S}'$ still given in (3.15) we can finally write

$$\Delta^2 = \Delta_1 * \Delta_2 \tag{3.20}$$

In contradistinction to the ordinary case, now $\overline{S}' \neq (S')^\dagger$. Moreover, the star product, when expanded, could in principle generate different terms in the two actions. Therefore, the chain of identities (3.16) is not immediately generalizable to the NAC case and we cannot identify $\Delta = \Delta_1$. However, working in antichiral representation, from the Feynman

rules it follows that the one-loop contributions to the effective action are actually equal, as we are now going to show. Therefore, in order to compute the effective action Δ we can indifferently concentrate on Δ_1 or Δ_2.

Following closely the ordinary case [26] we derive the Feynman rules from S' and $\overline{S}'$ by first extracting the quadratic part of the actions and then reading the vertices from the rest. Since the identities involving covariant derivatives are formally the same except that the products are now star products, the procedure to obtain the analytic expressions associated to the vertices is formally the same. We then refer the reader to Ref. [26] for details while reporting here only the final rules:

- Propagators

$$
\begin{aligned}
\langle \mathbf{\Phi}(z)\mathbf{\Phi}(z')\rangle &= -\frac{1}{\Box_0}\delta^{(8)}(z-z') \\
\langle \overline{\mathbf{\Phi}}(z)\overline{\mathbf{\Phi}}(z')\rangle &= -\frac{1}{\Box_0}\delta^{(8)}(z-z')
\end{aligned}
\tag{3.21}
$$

- Chiral vertices: At one loop the prescription for chiral superfields requires one to associate with one vertex

$$\frac{1}{2}(\overline{\boldsymbol{\nabla}}^2 - \overline{D}^2)D^2 \tag{3.22}$$

  and with the other vertices

$$\frac{1}{2}(\boldsymbol{\Box}_+ - \Box_0) \tag{3.23}$$

  where, with the definition $\overline{\boldsymbol{\nabla}}^2 D^2 \overline{\boldsymbol{\nabla}}^2 = \boldsymbol{\Box}_+ \overline{\boldsymbol{\nabla}}^2$

$$\boldsymbol{\Box}_+ = \boldsymbol{\Box}_{cov} - i\widetilde{\mathbf{W}}^\alpha * \boldsymbol{\nabla}_\alpha - \frac{i}{2}(\boldsymbol{\nabla}^\alpha * \widetilde{\mathbf{W}}_\alpha) \qquad , \qquad \boldsymbol{\Box}_{cov} = \frac{1}{2}\boldsymbol{\nabla}^{\alpha\dot{\alpha}} * \boldsymbol{\nabla}_{\alpha\dot{\alpha}} \tag{3.24}$$

- Antichiral vertices: The prescription is similar to the chiral case except that we associate $\frac{1}{2}D^2(\overline{\boldsymbol{\nabla}}^2 - \overline{D^2})$ at one vertex and $\frac{1}{2}(\boldsymbol{\Box}_- - \Box_0)$ at the other vertices with $D^2\overline{\boldsymbol{\nabla}}^2 D^2 = \boldsymbol{\Box}_- D^2$ or, explicitly,

$$\boldsymbol{\Box}_- = \boldsymbol{\Box}_{cov} - i\overline{\mathbf{W}}^{\dot{\alpha}} * \overline{\boldsymbol{\nabla}}_{\dot{\alpha}} - \frac{i}{2}(\overline{\boldsymbol{\nabla}}^{\dot{\alpha}} * \overline{\mathbf{W}}_{\dot{\alpha}}) \tag{3.25}$$

  These vertices are then expanded in powers of the background fields.

Now, consider the one-loop chiral contribution to the amplitude, obtained from (3.22, 3.23). Omitting irrelevant factors it can be written (recalling the definition of $\boldsymbol{\Box}_+$) as

$$\mathrm{Tr}\left\{\cdots\cdots(\overline{\boldsymbol{\nabla}}^2 - \overline{D}^2)D^2\frac{1}{\Box_0}(\overline{\boldsymbol{\nabla}}^2 - \overline{D}^2)D^2\frac{1}{\Box_0}(\overline{\boldsymbol{\nabla}}^2 - \overline{D}^2)D^2\frac{1}{\Box_0}\cdots\cdots\right\} \tag{3.26}$$

where the trace includes integrations and the propagators include the δ-functions. As in the usual supergraph rules the D^2 factors can be moved to the right across the propagators. Subsequently, at all but one of the vertices we can rewrite factors $D^2(\overline{\boldsymbol{\nabla}}-\overline{D}^2)$ as $(\boldsymbol{\square}_- - \square_0)$ and we immediately obtain the expression for the antichiral one-loop. Thus, we have shown that $\Delta = \Delta_1 = \Delta_2$ and it is sufficient to calculate one of the contributions. Because it is simpler in antichiral representation, we shall compute the contribution from the antichiral loop.

The procedure can be easily extended to the case of massive chirals by simply promoting the propagators (3.21) to massive propagators $-1/(\square_0 - m^2)$. We also note that these rules are strictly one-loop rules. At higher orders there are no difficulties and ordinary rules apply, as described in [26] with obvious modifications required by noncommutativity.

We can write down a formal effective interaction lagrangian that corresponds to the one-loop rules above. In the case of massive matter (chirals with mass m and antichirals with $\overline{m}$) in the adjoint representation of the gauge group, it is given by (from now on we avoid indicating star products and drop the boldface notation when no confusion arises)

$$\overline{S}_0 + \overline{S}_1 + \overline{S}_2 \equiv \int d^4x d^4\theta \ \mathrm{Tr} \left\{ \overline{\xi}(\square_0 - m\overline{m})\xi + \frac{1}{2}\left[\overline{\xi} D^2(\overline{\nabla}^2 - \overline{D}^2)\xi \ + \ \overline{\xi}(\overline{\square}_+ - \square_0)\xi \right] \right\} \tag{3.27}$$

where ξ , $\overline{\xi}$ are *unconstrained* quantum fields and the first vertex must appear once, and only once, in a one-loop diagram.

In terms of connections and field strengths and after some integration by parts we can rewrite it as

$$\begin{aligned}
\overline{S}_1 &= \int d^4x d^4\theta \ \mathrm{Tr} \left\{ \left(\frac{i}{4}\overline{\Gamma}^{\dot\alpha}[\xi, \overline{D}_{\dot\alpha} D^2 \overline{\xi}] - \frac{i}{4}\overline{\Gamma}^{\dot\alpha}[\overline{D}_{\dot\alpha}\xi, D^2\overline{\xi}] \right) + \left(-\frac{1}{4}\overline{\xi}\{\overline{\Gamma}^{\dot\alpha}[\overline{\Gamma}_{\dot\alpha}, D^2\xi]\} \right) \right\} \\
&\equiv \overline{S}_1 + \overline{S}_1' \qquad\qquad (3.28) \\
\overline{S}_2 &= \int d^4x d^4\theta \ \mathrm{Tr} \left\{ \left(\frac{i}{4}\overline{\Gamma}^{\alpha\dot\alpha}[\xi, \partial_{\alpha\dot\alpha}\overline{\xi}] - \frac{i}{4}\overline{\Gamma}^{\alpha\dot\alpha}[\partial_{\alpha\dot\alpha}\xi, \overline{\xi}] \right) + \left(\frac{i}{4}\overline{W}^{\dot\alpha}[\xi, \overline{D}_{\dot\alpha}\overline{\xi}] - \frac{i}{4}\overline{W}^{\dot\alpha}[\overline{D}_{\dot\alpha}\xi, \overline{\xi}] \right) \right. \\
&\qquad \left. + \ \left(\frac{1}{4}[\overline{\Gamma}^{\dot\alpha}, \xi][\overline{W}_{\dot\alpha}, \overline{\xi}] + \frac{1}{4}[\overline{W}^{\dot\alpha}, \xi][\overline{\Gamma}_{\dot\alpha}, \overline{\xi}] \right) + \left(\frac{1}{4}[\overline{\Gamma}^{\alpha\dot\alpha}, \xi][\overline{\Gamma}_{\alpha\dot\alpha}, \overline{\xi}] \right) \right\} \\
&\equiv \overline{S}_2 + \overline{S}_2' + \overline{S}_2'' + \overline{S}_2''' \qquad\qquad (3.29)
\end{aligned}$$

4 Feynman rules in momentum space

In this Section we describe the general procedure we follow to perform one–loop calculations. As in the ordinary supersymmetric theories, in general it is much more convenient to perform quantization and renormalization directly in superspace without going to components in the WZ gauge. In particular, this becomes unavoidable for NAC theories where the implications of nonanticommutativity on gauge invariance are nontrivial [21].

In the NAC case, as in the ordinary noncommutative case, it is also convenient to keep the star product implicit by delaying its expansion as much as possible. This can be accomplished by moving to a *momentum superspace* setup. Given a generic superfunction of superspace coordinates we Fourier transform both the bosonic and the fermionic coordinates according to the prescription

$$\widetilde{\Phi}(p,\pi,\overline{\pi}) = \int d^4x d^2\theta d^2\overline{\theta}\ e^{ipx+i\pi\theta+i\overline{\pi}\overline{\theta}}\ \Phi(x,\theta,\overline{\theta}) \tag{4.1}$$

In so doing the star products get traded for exponential factors dependent on the spinorial momentum variables

$$\Phi(x,\theta,\overline{\theta}) * \Psi(x,\theta,\overline{\theta}) \longrightarrow e^{\pi\wedge\pi'}\ \widetilde{\Phi}(p,\pi,\overline{\pi})\widetilde{\Psi}(p',\pi',\overline{\pi}') \tag{4.2}$$

where we have defined $\pi \wedge \pi' \equiv \pi_\alpha \mathcal{F}^{\alpha\beta}\pi'_\beta$.

We then develop perturbative techniques in momentum superspace. As for the ordinary anticommutative case this amounts to translating Feynman rules for propagators and vertices to momentum language. In particular, spinorial D derivatives become $\widetilde{D}$ derivatives according to the relations (B.8).

In the NAC case relevant changes occur due to the appearance of exponential factors at the vertices. In fact, given a local n–point vertex of the form $\int A_1 * \cdots * A_n$ the corresponding expression in momentum superspace becomes

$$\prod_{i<j}^{n} e^{\pi_i\wedge\pi_j}\ \widetilde{A}_1(\Pi_1)\cdots\widetilde{A}_n(\Pi_n)\ \delta^{(8)}(\sum_i \Pi_i) \tag{4.3}$$

where $\Pi_k \equiv (p_k, \pi_k, \overline{\pi}_k)$ denotes collectively the bosonic and fermionic momenta. When contracting quantum lines coming out from the vertices inside Feynman diagrams different ways of performing contractions lead to different configurations of exponential factors. Due to spinorial momentum conservation at each vertex the diagrams can be classified into *planar* diagrams characterized by exponentials depending only on the external momenta and *nonplanar* ones which have a nontrivial exponential dependence on the loop momenta. This pattern resembles closely what happens in the case of bosonic noncommutative theories [30, 31]. Therefore, we use the same prescriptions [30, 31] to determine the overall exponential factor associated with a given diagram.

The ordinary D–algebra which allows reducing a supergraph to an ordinary momentum diagram gets translated into a $\widetilde{D}$–algebra in a straightforward way. In particular, while in configuration superspace the general rule to get a nonzero contribution from a given supergraph is to perform D–algebra until we are left with a factor $D^2\overline{D}^2$ for each loop, in momentum superspace it gets translated into the requirement of performing $\widetilde{D}$–algebra until one ends up with a factor $\pi^2\overline{\pi}^2$ for each loop, where $(\pi,\overline{\pi})$ are the loop spinorial momenta. Therefore, once the exponential factor and the structure of the $\widetilde{D}$ derivatives associated to a given diagram have been established we proceed by performing $\widetilde{D}$–algebra.

This amounts to reducing the number of spinorial derivatives by using identities (B.11), expanding the exponential factors as

$$e^{\pi_1\wedge\pi_2} = 1 + \pi_1^\alpha \mathcal{F}_{\alpha\beta}\pi_2^\beta - \frac{1}{2}\pi_1^2\mathcal{F}^2\pi_2^2 \tag{4.4}$$

and selecting those configurations of spinorial momenta which have a factor $\pi^2\overline{\pi}^2$ for each loop. It is important to note that while $\overline{\pi}^2$ factors only come from $\widetilde{\overline{D}}$ derivatives associated to the vertices as in the ordinary case, π^2 factors can also come from the expansion (4.4), giving extra nonvanishing contributions to a given diagram proportional to the nonanticommutation parameter $\mathcal{F}$. This is the way nonanticommutativity enters the calculations in our approach.

Finally, once $\widetilde{D}$–algebra has been performed, we are left with ordinary momentum loop integrals. We evaluate them in dimensional regularization ($n = 4 - 2\epsilon$) and in the G–scheme in order to avoid dealing with irrelevant constants coming from the expansion of gamma functions.

We now list the propagators and the vertices in momentum superspace as used in our calculations. In order to simplify the notation, we omit the tildes over the Fourier transformed superfields and for a generic superfield Ψ we write $\Psi(k) \equiv \Psi(\Pi_k)$. When integration over superspace momenta is present, $\int$ stands for

$$\int \prod_{k=1}^{r} d^8\Pi_k \; \delta^{(8)}\left(\sum_{k=1}^{r}\Pi_r\right) \quad , \tag{4.5}$$

where r is the number of superfields appearing in the integral. Finally, we find it convenient to introduce the structures

$$\mathcal{P}^{ABC}(\pi_1,\pi_2) \equiv i\; f^{ABC}\cosh(\pi_1\wedge\pi_2) + d^{ABC}\sinh(\pi_1\wedge\pi_2) \tag{4.6}$$

$$\begin{aligned}\mathcal{Q}^{ABCD}(\pi_1,\pi_2,\pi_3,\pi_4) \;\equiv\;& \mathrm{Tr}([T^A,T^B][T^C,T^D])\;\cosh(\pi_1\wedge\pi_2)\;\cosh(\pi_3\wedge\pi_4)\\ +\;& \mathrm{Tr}(\{T^A,T^B\}[T^C,T^D])\;\sinh(\pi_1\wedge\pi_2)\;\cosh(\pi_3\wedge\pi_4)\\ +\;& \mathrm{Tr}([T^A,T^B]\{T^C,T^D\})\;\cosh(\pi_1\wedge\pi_2)\;\sinh(\pi_3\wedge\pi_4)\\ +\;& \mathrm{Tr}(\{T^A,T^B\}\{T^C,T^D\})\;\sinh(\pi_1\wedge\pi_2)\;\sinh(\pi_3\wedge\pi_4)\end{aligned} \tag{4.7}$$

arising from the star products at the vertices.

- <u>Quantum gauge superfields</u>

  Following the gauge–fixing procedure and the results discussed in Section 3 we can derive the Feynman rules for the pure gauge action (2.24). In Feynman gauge the V propagator reads

$$\langle V^A(1)\; V^B(2)\rangle = -g^2\delta^{AB}\,\frac{1}{p_1^2}\;\delta^{(8)}(\Pi_1+\Pi_2) \tag{4.8}$$

whereas the vertices useful for one-loop calculations are

$$
\begin{aligned}
S_{inv} + S_{GF} \rightarrow \quad & \frac{1}{2\, g^2} \int \mathcal{P}^{ABC}(\pi_2, \pi_3) \Big\{ V^A(1)\, \overline{\Gamma}^{\alpha\dot{\alpha}B}(2)\, p_{3\alpha\dot{\alpha}} V^C(3) \\
& + \frac{1}{2}\, V^A(1)\, p_{2\alpha\dot{\alpha}}\, \overline{\Gamma}^{\alpha\dot{\alpha}B}(2)\, V^C(3) + i\, V^A(1)\, \overline{W}^{\dot{\alpha}B}(2) \, \Big(\widetilde{\overline{D}}_{\dot{\alpha}}\, V^C(3)\Big) \\
& + i\, V^A(1)\, \widetilde{W}^{\alpha B}(2) \, \Big(\widetilde{D}_\alpha V^C(3)\Big) \Big\} \\
+ & \frac{1}{2\, g^2} \int \mathcal{Q}^{ABCD}(\pi_1, \pi_2, \pi_3, \pi_4) \Big\{ \frac{1}{2}\, V^A(1)\, \overline{\Gamma}^{\alpha\dot{\alpha}B}(2)\, \overline{\Gamma}^C_{\alpha\dot{\alpha}}(3)\, V^D(4) \\
& \qquad + \ V^A(1)\, \overline{W}^{\dot{\alpha}B}(2)\, \overline{\Gamma}^C_{\dot{\alpha}}(3)\, V^D(4) \Big\}
\end{aligned}
\tag{4.9}
$$

where the structures $\mathcal{P}$ and $\mathcal{Q}$ have been defined in eqs. (4.6) and (4.7).

- Quantum (anti)chiral superfields

  We now consider the matter/ghost actions and concentrate only on the parts which contribute to the pure gauge one–loop effective action. As discussed in the previous Section its structure can be read from the "effective" action (3.27) and it is the same for both chiral and antichiral sectors.

  The propagator for the unconstrained superfields ξ, $\overline{\xi}$ is (for ghosts we just set $m = \overline{m} = 0$)

$$
\langle \overline{\xi}^A(1) \xi^B(2) \rangle = \delta^{AB}\, \frac{1}{p_1^2 + m\overline{m}}\, \delta^{(8)}\left(\Pi_1 + \Pi_2\right) \tag{4.10}
$$

  whereas performing FT of the interaction terms we are interested in, we find cubic vertices

$$
\overline{S}_1 = \frac{i}{4} \int \mathcal{P}^{ABC}(\pi_1, \pi_2)\, \overline{\Gamma}^{\dot{\alpha}A}(1)\, [\ \xi^B(2)\, \widetilde{\overline{D}}_{\dot{\alpha}} \widetilde{D}^2 \overline{\xi}^C(3) - \widetilde{\overline{D}}_{\dot{\alpha}} \xi^B(2)\, \widetilde{D}^2 \overline{\xi}^C(3)\ ] \tag{4.11}
$$

$$
\overline{S}_2 = \frac{1}{4} \int \mathcal{P}^{ABC}(\pi_1, \pi_2)\, [\, -p_{2\alpha\dot{\alpha}} + p_{3\alpha\dot{\alpha}}]\, \overline{\Gamma}^{\alpha\dot{\alpha}A}(1)\, \xi^B(2)\, \overline{\xi}^C(3) \tag{4.12}
$$

$$
\overline{S}'_2 = \frac{i}{4} \int \mathcal{P}^{ABC}(\pi_1, \pi_2)\, \overline{W}^{\dot{\alpha}A}(1)\, [\ \xi^B(2)\, \widetilde{\overline{D}}_{\dot{\alpha}} \overline{\xi}^C(3) - \widetilde{\overline{D}}_{\dot{\alpha}} \xi^B(2)\, \overline{\xi}^C(3)\ ] \tag{4.13}
$$

  and quartic vertices

$$
\overline{S}'_1 = -\frac{1}{4} \int \mathcal{Q}^{ABCD}(\pi_1, \pi_2, \pi_3, \pi_4)\, \overline{\xi}^A(1)\, \overline{\Gamma}^{\dot{\alpha}B}(2)\, \overline{\Gamma}^C_{\dot{\alpha}}(3)\, \widetilde{D}^2 \xi^D(4) \tag{4.14}
$$

$$
\begin{aligned}
\overline{S}''_2 = -\frac{1}{4} \int & \mathcal{Q}^{ABCD}(\pi_1, \pi_2, \pi_3, \pi_4) \ \times \\
& \overline{\xi}^A(1) \Big(\overline{\Gamma}^{\dot{\alpha}B}(2)\, \overline{W}^C_{\dot{\alpha}}(3) + \overline{W}^{\dot{\alpha}B}(2)\, \overline{\Gamma}^C_{\dot{\alpha}}(3) \Big)\ \xi^D(4)
\end{aligned}
$$

$$(4.15)$$

$$\overline{S}_2''' = -\frac{1}{4}\int \mathcal{Q}^{ABCD}(\pi_1,\pi_2,\pi_3,\pi_4)\ \overline{\xi}^A(1)\ \overline{\Gamma}^{\alpha\dot{\alpha}B}(2)\ \overline{\Gamma}^C_{\alpha\dot{\alpha}}(3)\ \xi^D(4) \qquad (4.16)$$

where again $\mathcal{P}$ and $\mathcal{Q}$ are given in (4.6) and (4.7).

5 General structure of divergent terms

We begin by considering a SYM theory with or without matter described by the actions (3.4) or (2.24) respectively, and concentrate on the perturbative gauge effective action and its divergence structure. It is convenient to first investigate the general background dependence of possible divergent contributions. This can be accomplished by following the procedure of [10], [18], [11] based on dimensional analysis and invariance under global (pseudo)symmetries. This procedure has been used to constrain the structure of the NAC Wess-Zumino effective action, first in components [10] and then directly in superspace [18] by imposing the invariance under two $U(1)$ global (pseudo)symmetries. A similar analysis has been used in [11] for $N=\frac{1}{2}$ SYM theory in components, in the WZ gauge, by requiring invariance under a global R–symmetry. Here we apply the same analysis directly in superspace without fixing any supergauge. We assign suitable R-charges to the various quantities in the classical action and determine the most general dependence of a divergent term on the background superfields by requiring it to have null R-charge and a non-negative power dependence on the momentum UV cut–off Λ ¶.

The crucial observation is the following: since we work in antichiral representation any superfield appearing in the final structure of a divergent term can be written in terms of $\overline{\Gamma}^{\dot{\alpha}}$ (see discussion in Section 3). Moreover, since the general counterterm has to be $N=1/2$ supersymmetric its structure in superspace might contain explicit dependence on powers of $\overline{\theta}$. For simplicity, we forget about spinorial and color indices. With the following assignment of R–charges

¶For this purpose we find more convenient to work with a cut–off regularization rather than dimensional regularization. In any case the selection of possible divergent structures is independent of the regularization used.

	dim	R-charge
$\overline{\Gamma}^{\dot{\alpha}}$	$1/2$	-1
D_α	$1/2$	1
$\overline{D}_{\dot{\alpha}}$	$1/2$	-1
$\overline{\theta}$	$-1/2$	1
$\partial_{\alpha\dot{\alpha}}$	1	0
$\mathcal{F}^{\rho\gamma}$	-1	-2
Λ	1	0

the most general local term will have the form

$$\int d^4x\, d^4\theta\, \overline{\theta}^{\overline{\tau}}\, \Lambda^\beta\, \mathcal{F}^\alpha\, D^\gamma\, \overline{D}^{\overline{\gamma}}\, \partial^\delta\, \overline{\Gamma}^{\overline{\sigma}} \tag{5.1}$$

where $\overline{\tau}, \beta, \alpha, \gamma, \overline{\gamma}, \delta, \overline{\sigma}$ are all non-negative integers satisfying

$$\beta = 2 + \alpha - \frac{1}{2}(\gamma + \overline{\gamma}) - \delta - \frac{1}{2}\overline{\sigma} + \frac{1}{2}\overline{\tau} \tag{5.2}$$

from the condition that the integrand have mass dimension four and

$$\overline{\sigma} = \gamma - \overline{\gamma} - 2\alpha + \overline{\tau} \tag{5.3}$$

from requiring the vanishing of the total R-charge. In (5.1) ∂^δ stands for spacetime derivatives and gives the maximal power of momenta present in the counterterm.

Since we are looking for divergent terms we impose $\beta \geq 0$. Replacing α in (5.2) with its expression as obtained from eq. (5.3) we have

$$\overline{\sigma} \leq 2 + \overline{\tau} - \overline{\gamma} - \delta \tag{5.4}$$

Note that $\overline{\sigma}$ counts the number of background superfields in (5.1). It then gives the maximal number of gauge vertices present in the corresponding diagram. Therefore, from the condition (5.4) and from the observation that $\overline{\tau} \leq 2$ it follows that n-point functions with $n \geq 5$ always give convergent contributions and we can concentrate on two, three and four point functions when computing the divergent part of the effective action.

Pushing the analysis a bit further we find an extra constraint on $\overline{\sigma}$. Since $D^3 = 0$ and, having already exhibited the whole ∂–dependence, $\{D, \overline{D}\} \sim 0$, the maximal number of superspace covariant derivatives in (5.1) must satisfy

$$\gamma \leq 2\overline{\sigma} - 2 \tag{5.5}$$

It follows that $\gamma - \overline{\gamma} + \overline{\tau} \leq 2\overline{\sigma}$ and from (5.3) we have

$$\alpha \leq \frac{1}{2}\overline{\sigma} \leq 2 \tag{5.6}$$

This allows a complete classification of all possible divergent contributions. For $\alpha = 0$, only 2,3,4–point diagrams would be divergent. However, since they correspond to the divergences of the ordinary $\mathcal{N} = 1$ SYM theory [26, 32], 3,4–point divergent diagrams are ruled out by the restored $\mathcal{N} = 1$ supersymmetry ($\overline{\tau} = 0$) and we are left only with nonvanishing 2–point divergent contributions. In this case we know that all the terms are logarithmically divergent since supersymmetry prevents higher divergences to be generated.

In the NAC case new divergent terms proportional to the NAC parameter appear for

$$\begin{array}{llll} \bullet & \alpha = 1 & \rightarrow & \overline{\sigma} = 2,3,4 \\ \bullet & \alpha = 2 & \rightarrow & \overline{\sigma} = 4 \end{array} \tag{5.7}$$

We now list all possible divergences proportional to $\mathcal{F}$ ($\alpha = 1$) and $\mathcal{F}^2$ ($\alpha = 2$).

- $\alpha = 1$ $\overline{\sigma} = 2$

  In this case we have

$$\gamma - \overline{\gamma} + \overline{\tau} = 4, \qquad \gamma \leq 2 \qquad \rightarrow \qquad \gamma = 2, \;\; \overline{\gamma} = 0, \;\; \overline{\tau} = 2 \qquad \delta \leq 2 \tag{5.8}$$

  Therefore possible divergent terms are proportional to

$$\mathcal{F}^{\alpha}_{\;\;\alpha} \int d^4x \; d^4\theta \; \overline{\theta}^2 \; \overline{\Gamma}^{\dot{\alpha}} \; D^2\overline{\Gamma}_{\dot{\alpha}} \tag{5.9}$$

$$\mathcal{F}^{\alpha\beta} \int d^4x \; d^4\theta \; \overline{\theta}^2 \; (\partial_{\alpha}^{\;\;\dot{\alpha}} \; \overline{\Gamma}_{\dot{\alpha}}) \; (\partial_{\beta}^{\;\;\dot{\beta}} \; D^2\overline{\Gamma}_{\dot{\beta}}) \tag{5.10}$$

  However, both these expressions vanish for symmetry reasons independently of the color structure, as a consequence of the fact that $\mathcal{F}$ is symmetric in its spinorial indices ($\mathcal{F}^{\alpha}_{\;\;\alpha} \equiv \mathcal{F}^{\alpha\beta}C_{\beta\alpha} = 0$ and $\mathcal{F}^{\alpha\beta}$ contracted with two identical spinors vanishes).

- $\alpha = 1$ $\overline{\sigma} = 3$

  It follows that

$$\gamma - \overline{\gamma} + \overline{\tau} = 5, \qquad \gamma \leq 4 \tag{5.11}$$

  Therefore we have three cases:

$$\gamma = 4, \;\; \overline{\tau} = 2, \;\; \overline{\gamma} = 1, \;\; \delta = 0 \quad \rightarrow \quad \mathcal{F}^{\alpha}_{\;\;\alpha} \int d^4x \; d^4\theta \; \overline{\theta}^2 \;\; \overline{D}^{\dot{\alpha}}\overline{\Gamma}_{\dot{\alpha}} \; D^2\overline{\Gamma}^{\dot{\beta}} \; D^2\overline{\Gamma}_{\dot{\beta}} \tag{5.12}$$

$$\gamma = 4, \quad \overline{\tau} = 1, \quad \overline{\gamma} = 0, \quad \delta = 0 \quad \rightarrow \quad \mathcal{F}^{\alpha}_{\ \alpha} \int d^4x \; d^4\theta \; \overline{\theta}^{\dot{\alpha}} \;\; \overline{\Gamma}_{\dot{\alpha}} \; D^2\overline{\Gamma}^{\dot{\beta}} \; D^2\overline{\Gamma}_{\dot{\beta}} \tag{5.13}$$

$$\gamma = 3, \quad \overline{\tau} = 2, \quad \overline{\gamma} = 0, \quad \delta = 1 \quad \rightarrow \quad \mathcal{F}^{\alpha\beta} \int d^4x \; d^4\theta \; \overline{\theta}^2 \;\; \partial_{\alpha\dot{\alpha}} D_\beta \overline{\Gamma}^{\dot{\alpha}} \; \overline{\Gamma}^{\dot{\beta}} \; D^2\overline{\Gamma}_{\dot{\beta}} \tag{5.14}$$

Again, the first two structures vanish for symmetry reasons, while the third one can survive together with similar expressions which differ in the contractions of spinorial indices and position of the spacetime derivative. We note that any nontrivial trace structure is allowed.

- $\alpha = 1 \;\; \overline{\sigma} = 4$

  In this case we have

$$\gamma - \overline{\gamma} + \overline{\tau} = 6, \quad \gamma \leq 6, \quad \overline{\tau} - \delta - \overline{\gamma} \geq 2 \quad \rightarrow \quad \delta = \overline{\gamma} = 0, \quad \gamma = 4 \;\; \overline{\tau} = 2 \tag{5.15}$$

  and all possible divergent terms are proportional to

$$\mathcal{F}^{\alpha\beta} \int d^4x \; d^4\theta \; \overline{\theta}^2 \; \overline{\Gamma}^{\dot{\alpha}} \; D_\alpha \overline{\Gamma}_{\dot{\alpha}} \; D_\beta \overline{\Gamma}^{\dot{\beta}} \; D^2\overline{\Gamma}_{\dot{\beta}} \tag{5.16}$$

  and similar expressions obtained by changing the contraction of the indices. Again, all possible nontrivial trace structures are allowed.

- $\alpha = 2 \;\; \overline{\sigma} = 4$

  We have

$$\gamma - \overline{\gamma} + \overline{\tau} = 8, \quad \gamma \leq 6 \quad \overline{\tau} - \delta - \overline{\gamma} \geq 2 \quad \rightarrow \quad \delta = \overline{\gamma} = 0, \quad \gamma = 6 \;\; \overline{\tau} = 2 \tag{5.17}$$

  Therefore all possible divergent terms are proportional to

$$\mathcal{F}^2 \int d^4x \; d^4\theta \; \overline{\theta}^2 \; \overline{\Gamma}^{\dot{\alpha}} \; D^2\overline{\Gamma}_{\dot{\alpha}} \; D^2\overline{\Gamma}^{\dot{\beta}} \; D^2\overline{\Gamma}_{\dot{\beta}} \tag{5.18}$$

  and similar expressions.

We note that in selecting possible divergent structures we did not take into account the requirement of supergauge invariance. For example, structures like the ones appearing in (5.14) are not gauge invariant on their own. As already discussed in [21] and in Section 2, these terms are required to appear in suitable linear combinations in order to guarantee supergauge invariance. Therefore, the previous analysis together with the requirement of supergauge invariance allow us to conclude that in performing loop calculations we can

focus only on the usual divergent diagrams for the undeformed theory and on diagrams which give rise to background structures of the form

$$\mathcal{F}^{\rho\gamma} \int d^4x \; d^4\theta \; \overline{\theta}^2 \; \mathrm{Tr}\left(\partial_{\rho\dot{\rho}}\overline{\Gamma}^{\dot{\alpha}}\right) \mathrm{Tr}\left(\overline{W}_{\dot{\alpha}}\overline{\Gamma}_{\gamma}^{\;\dot{\rho}}\right) \tag{5.19}$$

$$\mathcal{F}^{2} \int d^4x \; d^4\theta \; \overline{\theta}^2 \; \mathrm{Tr}\left(\overline{\Gamma}^{\dot{\alpha}}\overline{W}_{\dot{\alpha}}\overline{W}^{\dot{\beta}}\overline{W}_{\dot{\beta}}\right) \tag{5.20}$$

$$\mathcal{F}^{2} \int d^4x \; d^4\theta \; \overline{\theta}^2 \; \mathrm{Tr}\left(\overline{\Gamma}^{\dot{\alpha}}\overline{W}_{\dot{\alpha}}\right) \mathrm{Tr}\left(\overline{W}^{\dot{\beta}}\overline{W}_{\dot{\beta}}\right) \tag{5.21}$$

$$\mathcal{F}^{2} \int d^4x \; d^4\theta \; \overline{\theta}^2 \; \mathrm{Tr}\left(\overline{\Gamma}^{\dot{\alpha}}\right) \mathrm{Tr}\left(\overline{W}_{\dot{\alpha}}\right) \mathrm{Tr}\left(\overline{W}^{\dot{\beta}}\overline{W}_{\dot{\beta}}\right) \tag{5.22}$$

The 4–point structures (5.20-5.22) are supergauge invariant, while the 3-point function (5.19) is not. However, we include it in the list of "good" terms since, as already noted, it is the completion of the gauge–variant 2-point function $\int d^4x \; d^4\theta \; \mathrm{Tr}(\overline{\Gamma}^{\dot{\alpha}})\mathrm{Tr}(\overline{W}_{\dot{\alpha}})$.

Other 4-point terms proportional to $\mathcal{F}^2$ but with different color structures have not been included since they are identically zero for spinorial reasons. Finally, there are not 4-point functions of order $\mathcal{F}$ which satisfy the requirement of supergauge invariance.

Finally, a very important conclusion can be drawn from the previous analysis concerning the nature of the divergences. For values of the constants $\alpha, \overline{\tau}, \gamma, \overline{\gamma}, \delta, \overline{\sigma}$ corresponding to divergent nonvanishing structures the power β of the cutoff Λ as computed from (5.2) is always zero. Terms which would allow for values of β strictly positive actually vanish for symmetry reasons. Therefore, we have a direct proof in superspace that in $N = 1/2$ SYM theories the divergences continue to be only *logarithmic* as in the ordinary anticommutative case (A similar conclusion can be reached by working in components [11, 19]). This means that, in spite of the supersymmetry breaking induced by the $\mathcal{F}^{\alpha\beta}$ tensor the deformed theories maintain the nice quantum properties of the ordinary ones. The NAC mechanism of supersymmetry breaking can be considered a *soft* mechanism.

6 One–loop diagrams in the background field method

We begin by considering a SYM theory with matter in the adjoint representation of the $U(\mathcal{N})$ gauge group. As discussed in Sections 2,3 we find it convenient to work in gauge antichiral representation where the classical action is (3.4) $^{\|}$. Following the prescription described in Section 3 we perform quantum–background splitting, we add gauge–fixing terms and we end up with a quantized action whose Feynman rules suitable for one–loop calculations are summarized at the end of Section 3. The diagrams are then computed

$^{\|}$In Ref. [21] the divergent part of the one–loop gauge effective action was computed by working in a mixed chiral/antichiral setting for supergauge covariant derivatives. Here we reproduce that calculation in completely antichiral representation.

using momentum superspace techniques as described in Section 4. Besides the divergent contributions already present in the ordinary case, i.e. a gauge two–point function with a chiral loop [29, 26], we find new divergent contributions up to 4–point functions proportional to $\mathcal{F}$ and $\mathcal{F}^2$ due to nontrivial modifications to the D–algebra induced by the star product.

Calculations are performed in dimensional regularization ($n = 4-2\epsilon$) and the integrals useful for our goal are listed in Appendix A. In particular, all the divergences are expressed in terms of a self–energy integral $\mathcal{S}$ defined in (A.8).

We now list the divergent nontrivial contributions at one loop. Up to an overall divergent factor $\mathcal{S}$, they are:

- Ordinary terms: two–point function

  Following standard D-algebra arguments, ordinary one–loop divergent contributions with gauge external fields come only from the diagram in Fig.1 with a chiral matter/ghost quantum loop.

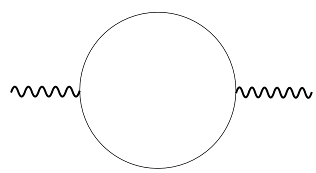

Figure 1: Gauge one–loop two–point functions from chiral loop.

  In antichiral representation it reads

$$\Gamma_2^{(1)} = \frac{1}{2}(-3 + N_f) \int d^4x \; d^4\theta \; \Big[\, \mathcal{N} \; \mathrm{Tr} \left(\overline{\Gamma}^{\dot{\alpha}} \; \overline{W}_{\dot{\alpha}} \right) - \mathrm{Tr} \left(\overline{\Gamma}^{\dot{\alpha}} \right) \mathrm{Tr} \left(\overline{W}_{\dot{\alpha}} \right) \Big] \tag{6.1}$$

  Here N_f counts the number of matter flavors while (-3) is the contribution from the three ghosts in the action (3.5).

- NAC terms: three– and four–point functions

  Three– and four–point diagrams with vector loops are all finite, whereas divergent contributions arise from the chiral loop diagrams in Figs. 2 and 3 for the three–point and four–point functions, respectively.

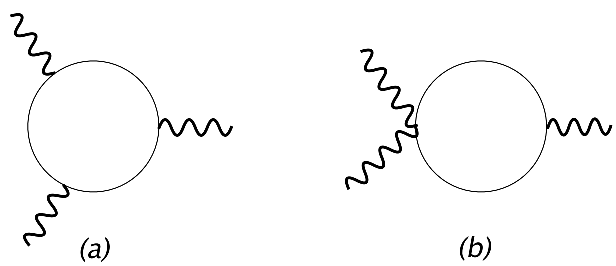

Figure 2: One–loop three–point functions with chiral loop.

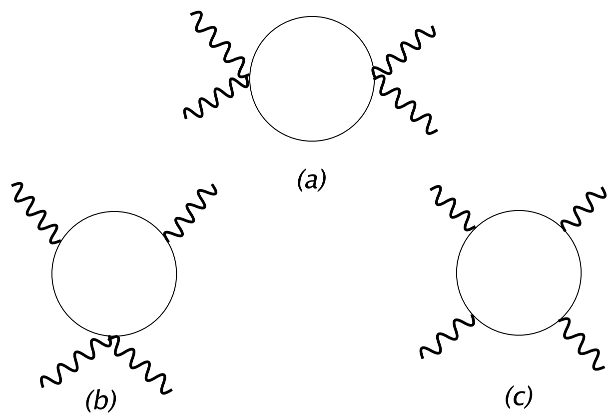

Figure 3: One–loop four–point functions with chiral loop.

In antichiral representation they read

$$\Gamma_3^{(1)} = -2i(-3+N_f)\mathcal{F}^{\rho\gamma}\int d^4x\ d^4\theta\ \ \overline{\theta}^2\ \mathrm{Tr}\left(\partial_{\rho\dot\rho}\overline{\Gamma}^{\dot\alpha}\right)\mathrm{Tr}\left(\overline{W}_{\dot\alpha}\overline{\Gamma}_{\gamma}^{\ \dot\rho}\right) \qquad (6.2)$$

$$\Gamma_4^{(1)} = \frac{1}{2}(-3+N_f)\mathcal{F}^2\int d^4x\ d^4\theta\ \ \overline{\theta}^2\ \mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\overline{W}_{\dot\alpha}\right)\mathrm{Tr}\left(\overline{W}^{\dot\beta}\overline{W}_{\dot\beta}\right) \qquad (6.3)$$

We note in both contributions the explicit dependence on $\overline{\theta}^2$ which indicates the partial breaking of supersymmetry.

To summarize, starting from the classical action (3.4) the one-loop divergent contributions to the gauge effective action have the form

$$\begin{aligned}\Gamma_{gauge}^{(1)} \ \ \rightarrow\ \ \frac{1}{2}(-3+N_f)\times\Big\{\int d^4x\ d^4\theta\ &\left[\mathcal{N}\mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\ \overline{W}_{\dot\alpha}\right)-\mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\right)\mathrm{Tr}\left(\overline{W}_{\dot\alpha}\right)\right]\\ &-4i\mathcal{F}^{\rho\gamma}\int d^4x\ d^4\theta\ \ \overline{\theta}^2\ \mathrm{Tr}\left(\partial_{\rho\dot\rho}\overline{\Gamma}^{\dot\alpha}\right)\mathrm{Tr}\left(\overline{W}_{\dot\alpha}\overline{\Gamma}_{\gamma}^{\ \dot\rho}\right)\\ &+\mathcal{F}^2\int d^4x\ d^4\theta\ \ \overline{\theta}^2\ \mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\overline{W}_{\dot\alpha}\right)\mathrm{Tr}\left(\overline{W}^{\dot\beta}\overline{W}_{\dot\beta}\right)\Big\} \qquad (6.4)\end{aligned}$$

As proven in [21] this result is supergauge invariant; while the first and the last terms are invariant on their own, the gauge variance of the third term compensates that of the abelian noninvariant $\mathrm{Tr}(\overline{\Gamma}^{\dot{\alpha}})\mathrm{Tr}(\overline{W}_{\dot{\alpha}})$ term as discussed in Section 2.

Given the result (6.4) for the divergent part of the effective action, we can immediately conclude that the action (3.4) is *not* one–loop renormalizable in a superspace setup. In fact, given the classical gauge action in (3.4) we cannot cancel by multiplicative renormalization the new divergent structures which arise at one loop.

It may be useful to compare our result with the one obtained in components, in the Wess-Zumino gauge [12]. This is made possible by the fact that (super)gauge invariance of $N = \frac{1}{2}$ SYM theories is not spoiled by quantum corrections [21]. Therefore, quantum properties like renormalizability do not depend on the particular (super)gauge and we can fix a particular one in order to simplify the study. In Appendix C we choose the WZ gauge and reduce the one–loop effective action (6.4) to components. This reduction confirms at the component level the result already evident in superspace, i.e. that the $N = \frac{1}{2}$ SYM theory defined by the action (3.4) as obtained by the natural deformation of the ordinary $N = 1$ SYM theory, is not one–loop renormalizable. Our result agrees with what has been found in [12].

7 The modified action

Given that the gauge action (2.24) is not renormalizable, the only way out is to modify the classical action from the very beginning by adding new terms which allow for the cancellation of all the divergent terms at one loop. This approach has been already applied to the theory in components in [12], where the one–loop renormalizable deformation in components has been found.

Our aim is to do the same in the more general superspace setup. We focus on the pure gauge deformed SYM theory (a brief discussion in the presence of matter superfields will be given in section 10) and start with a modified classical action which contains extra pieces up to quartic order in the superfields which are allowed by the symmetries of the theory

$$
\begin{aligned}
S_{final} = &\ \frac{1}{2\, g^2} \int d^4x\ d^4\theta\ \mathrm{Tr}\left(\overline{\Gamma}^{\dot{\alpha}} * \overline{W}_{\dot{\alpha}}\right) \\
&+ \frac{1}{2\, g_0^2\, \mathcal{N}} \int d^4x\ d^4\theta \Big[\ \mathrm{Tr}\left(\overline{\Gamma}^{\dot{\alpha}}\right) * \mathrm{Tr}\left(\overline{W}_{\dot{\alpha}}\right) \\
&\qquad\qquad + 4i\mathcal{F}^{\rho\gamma}\ \overline{\theta}^2\ \mathrm{Tr}\left(\partial_{\rho\dot{\rho}}\overline{\Gamma}^{\dot{\alpha}}\right) * \mathrm{Tr}\left(\overline{W}_{\dot{\alpha}} * \overline{\Gamma}_{\gamma}^{\ \dot{\rho}}\right)\Big] \\
&+ \frac{1}{2\, h^2\, \mathcal{N}}\mathcal{F}^2 \int d^4x\ d^4\theta\ \ \overline{\theta}^2\ \mathrm{Tr}\left(\overline{\Gamma}^{\dot{\alpha}} * \overline{W}_{\dot{\alpha}}\right) * \mathrm{Tr}\left(\overline{W}^{\dot{\beta}} * \overline{W}_{\dot{\beta}}\right)
\end{aligned}
$$

$$+\frac{1}{l^2}\,\mathcal{F}^2\int d^4x\; d^4\theta\;\overline{\theta}^2\mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}*\overline{W}_{\dot\alpha}*\overline{W}^{\dot\beta}*\overline{W}_{\dot\beta}\right)$$
$$+\frac{1}{r^2}\,\mathcal{F}^2\int d^4x\; d^4\theta\;\overline{\theta}^2\mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\right)*\mathrm{Tr}\left(\overline{W}_{\dot\alpha}\right)*\mathrm{Tr}\left(\overline{W}^{\dot\beta}*\overline{W}_{\dot\beta}\right) \tag{7.1}$$

Five coupling constants g, g_0, h, l, r have been introduced. The coefficients of the various terms have been chosen in such a way that if $g^2 = -g_0^2 = h^2$ (and setting $1/l^2 = 1/r^2 = 0$), we maintain the original ratio among the divergent terms coming from chiral/ghost loops (see eq. (6.4)).

The number of independent coupling constants is dictated by supergauge invariance. In fact, as already noted, while the g, h, l, and r terms are separately supergauge invariant, the terms proportional to g_0 have gauge variations which cancel only if the ratio between their coefficients is the one given in (2.22).

We note that the new action (7.1) is written with $*$-product between the superfields. However we can replace it with the ordinary product without producing new terms.

8 The modified action: One–loop diagrams

We consider the case of a NAC gauge theory with no matter, now described by the classical action (7.1) and address the issue of its quantization.

We perform quantum–background splitting and expand each term in (7.1) to second order in the quantum fields. In this manner we generate the kinetic terms for the gauge superfields and vertices of the form quantum–quantum–background, the only vertices which can contribute at one loop.

The first question concerns the gauge–fixing procedure and, in particular, the choice of a convenient gauge–fixing function and the corresponding gaussian weight factor in the functional integral. In Appendix D we treat this problem in all detail whereas here we briefly summarize the main points. The introduction of a new quadratic term proportional to the coupling constant g_0 assigns different weights to the quadratic parts of the abelian $U(1)$ and non–abelian vector fields. As discussed in Appendix D, in the ordinary anticommutative case, given gauge–fixing functions suitable for the background field method we have different possibilities for the choice of the gaussian weight factor in the functional integral (see eqs. (D.1) and (D.7)). In particular, choosing the gaussian factor (D.7) would lead to the same expression for the propagators of the abelian and non–abelian fields, so in the ordinary case it might be the most convenient choice, whereas the choice (D.1) would lead to different propagators for the $U(1)$ and $SU(\mathcal{N})$ fields. However, in the presence of the $*$-product the first choice becomes inappropriate when working in background field method since it generates a quadratic term in the action which is not background gauge invariant. Therefore, for our purposes it is preferable to choose a gauge–fixing procedure which preserves background gauge invariance but leads

to different propagators for the $SU(\mathcal{N})$ and $U(1)$ vector superfields as given in (D.8) and (D.11). In momentum superspace they read

$$\langle V^a(1)\; V^b(2)\rangle = -\delta^{ab}\, \frac{g^2}{p_1^2}\, \delta^{(8)}\left(\Pi_1 + \Pi_2\right)$$
$$\langle V^0(1)\; V^0(2)\rangle = -\frac{g^2}{p_1^2}\, \left[1 - \left(\frac{g^2}{g_0^2 + g^2}\right) \frac{\widetilde{\overline{D}}^{\dot{\alpha}}\, \widetilde{D}^2 \widetilde{\overline{D}}_{\dot{\alpha}}}{p_1^2}\right] \delta^{(8)}\left(\Pi_1 + \Pi_2\right) \qquad (8.1)$$

Feynman rules for the vertices are still the ones given at the end of Section 3 supplemented by new vertices coming from the extra pieces in (7.1). These new vertices are listed in Appendix E. It is easy to realize that at this order the new vertices will enter only diagrams with vector fields inside the loop. Therefore the contributions from the chiral loops, which in this case come only from ghosts, are not modified and can be read from Section 6. Thus, the ghost contribution to the divergent part of the effective action is (6.4) with $N_f = 0$.

We then concentrate on the evaluation of the new divergent diagrams built up with the new vertices in Appendix E for the modified action. In contradistinction to the vertices coming from (2.24) which never enter one–loop divergent diagrams (see discussion in Section 6 and [21]), the new vertices can give rise to many new divergent contributions. Despite the possibility of having more than one hundred divergent diagrams, we can use the general arguments in Section 5 as a way to eliminate from the very beginning diagrams which would not lead to the correct structure. Moreover, superspace techniques allow a straightforward cancellation of most of the remaining ones and the final list of nontrivial diagrams is quite contained.

In what follows we report the results for the new divergent diagrams.

- <u>Three–point functions</u>
  The contributions to the three–point function are described by the diagrams in Fig.4, but they all *cancel out* in a nontrivial way.

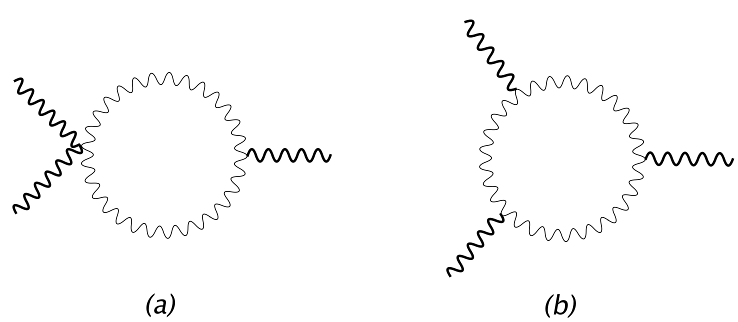

Figure 4: Gauge one–loop three–point functions from vector loop.

We would have expected this result for the following reason: from gauge invariance considerations [21] any one–loop correction has to be gauge invariant. However, there is no way to build a gauge invariant three-point function proportional to $\mathcal{F}$ unless it comes together with a suitable two–point function as given in (2.22). On the other hand, we know from Section 5 that divergent two–point functions proportional to $\mathcal{F}$ cannot be produced: In superspace, only ordinary $\int d^4\theta\ \mathrm{Tr}(\overline{\Gamma}^{\dot\alpha}\overline{W}_{\dot\alpha})$ and $\int d^4\theta\ \mathrm{Tr}(\overline{\Gamma}^{\dot\alpha})\mathrm{Tr}(\overline{W}_{\dot\alpha})$ can appear, and these corrections arise only from chiral loop diagrams (not vectors ones!) for D–algebra reasons. Therefore, the absence of a two–point divergent contribution from vector loops rules out the possibility of having a nonvanishing three–point correction.

- Four–point functions
  The contributions to the four–point functions come from the diagrams in Fig.5

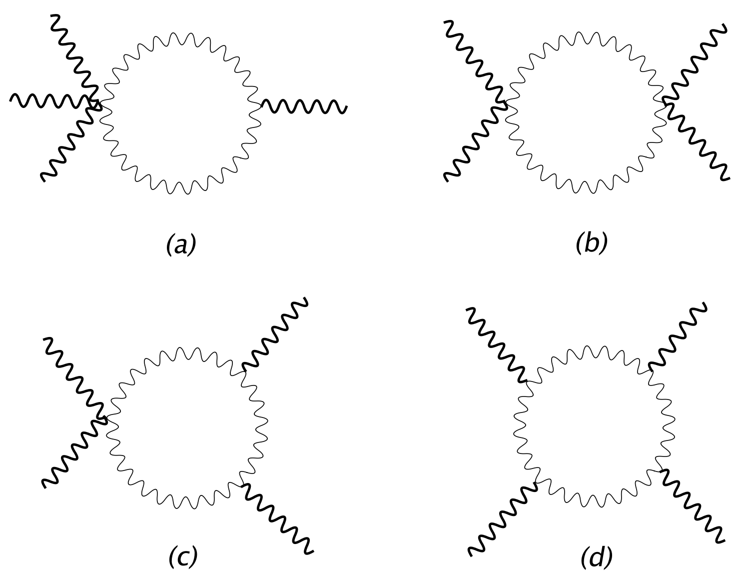

Figure 5: Gauge one–loop four–point functions from vector loop.

Even if in this case there is no complete cancellation, most of the possible divergent diagrams give null contributions due to standard superspace D–algebra. Moreover, it is easy to see that diagrams with the structure in Fig. 5(b) cancel among themselves. Therefore, summing all the remaining terms, we have four kinds of contributions with the same trace structure but depending on four different combinations of coupling constants

$$\text{Contribution} \sim \frac{\widetilde{g}^2}{g_0^2\,\mathcal{N}} \;:\qquad -24\mathcal{F}^2\int d^4x\ d^4\theta\ \overline{\theta}^2\mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\ \overline{W}_{\dot\alpha}\ \overline{W}^{\dot\beta}\ \overline{W}_{\dot\beta}\right)$$
$$\text{Contribution} \sim \frac{g^2}{h^2\,\mathcal{N}} \;:\qquad -12\mathcal{F}^2\int d^4x\ d^4\theta\ \overline{\theta}^2\mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\ \overline{W}_{\dot\alpha}\ \overline{W}^{\dot\beta}\ \overline{W}_{\dot\beta}\right)$$

$$\text{Contribution} \sim \frac{g^2\widetilde{g}^2}{g_0^4\,\mathcal{N}} : \qquad -12\mathcal{F}^2 \int d^4x\; d^4\theta\; \overline{\theta}^2 \mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\; \overline{W}_{\dot\alpha}\; \overline{W}^{\dot\beta}\; \overline{W}_{\dot\beta}\right)$$
$$\text{Contribution} \sim \frac{g^2\mathcal{N}}{l^2} : \qquad -\,6\mathcal{F}^2 \int d^4x\; d^4\theta\; \overline{\theta}^2\; \mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\; \overline{W}_{\dot\alpha}\; \overline{W}^{\dot\beta}\; \overline{W}_{\dot\beta}\right)$$

where $\widetilde{g}^2$ is defined in equation (D.10).

We note that terms of the form $\int d^4\theta\; \overline{\theta}^2 \mathrm{Tr}(\overline{\Gamma}^{\dot\alpha}\overline{W}_{\dot\alpha})\mathrm{Tr}(\overline{W}^{\dot\beta}\; \overline{W}_{\dot\beta})$ and $\int d^4\theta\; \overline{\theta}^2 \mathrm{Tr}(\overline{\Gamma}^{\dot\alpha})\mathrm{Tr}(\overline{W}^{\dot\beta})\; \mathrm{Tr}(\overline{W}_{\dot\beta}\; \overline{W}_{\dot\alpha})$ cancel along the calculation.

Finally, adding to the classical action all the divergent contributions from ghost and vector loops to the one-loop gauge effective action we obtain

$$\begin{aligned}
\Gamma^{(1)} = {}& \frac{1}{2\; g^2} \int d^4x\; d^4\theta\; \mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\overline{W}_{\dot\alpha}\right) \times \left[1 - 3\frac{g^2\; \mathcal{N}}{(4\pi)^2\; \epsilon}\right] \\
&+ \frac{1}{2\; g_0^2\; \mathcal{N}} \int d^4x\; d^4\theta \left[\mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\right) \mathrm{Tr}\left(\overline{W}_{\dot\alpha}\right)\right. \\
&\qquad\qquad \left. + 4i\mathcal{F}^{\rho\gamma}\overline{\theta}^2\; \mathrm{Tr}\left(\partial_{\rho\dot\rho}\overline{\Gamma}^{\dot\alpha}\right) \mathrm{Tr}\left(\overline{W}_{\dot\alpha}\overline{\Gamma}_{\gamma}^{\;\dot\rho}\right)\right] \times \left[1 + 3\frac{g_0^2\; \mathcal{N}}{(4\pi)^2\; \epsilon}\right] \\
&+ \frac{1}{2\; h^2\; \mathcal{N}}\mathcal{F}^2 \int d^4x\; d^4\theta\;\; \overline{\theta}^2\; \mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\overline{W}_{\dot\alpha}\right) \mathrm{Tr}\left(\overline{W}^{\dot\beta}\overline{W}_{\dot\beta}\right) \times \left[1 - 3\frac{h^2\; \mathcal{N}}{(4\pi)^2\; \epsilon}\right] \\
&+ \frac{1}{l^2}\; \mathcal{F}^2 \int d^4x\; d^4\theta\; \overline{\theta}^2\; \mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\; \overline{W}_{\dot\alpha}\; \overline{W}^{\dot\beta}\; \overline{W}_{\dot\beta}\right) \times \\
&\qquad\qquad \left[1 - 6\frac{g^2}{(4\pi)^2\; \epsilon}\left(\mathcal{N} + \frac{2l^2}{h^2\; \mathcal{N}} + \left(2 + \frac{g^2}{g_0^2}\right)\frac{2l^2}{\mathcal{N}\,(g_0^2 + g^2)}\right)\right] \\
&+ \frac{1}{r^2}\; \mathcal{F}^2 \int d^4x\; d^4\theta\; \overline{\theta}^2\; \mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\right) \mathrm{Tr}\left(\overline{W}_{\dot\alpha}\right) \mathrm{Tr}\left(\overline{W}^{\dot\beta}\; \overline{W}_{\dot\beta}\right)
\end{aligned} \tag{8.2}$$

Since the divergent terms have the same background structure as the classical modified action we expect the theory described by (7.1) to be at least one–loop renormalizable. This will be the subject of the next Section.

9 Renormalization and β-functions

We now proceed to the renormalization of the theory. We define renormalized coupling constants as **

$$\begin{aligned}
&g = \mu^{-\epsilon} Z_g^{-1} g_{(B)} && g_0 = \mu^{-\epsilon} Z_{g_0}^{-1} g_{0(B)} && \\
&h = \mu^{-\epsilon} Z_h^{-1} h_{(B)} && l = \mu^{-\epsilon} Z_l^{-1} l_{(B)} && r = \mu^{-\epsilon} Z_r^{-1} r_{(B)}
\end{aligned} \tag{9.1}$$

**in this setup there is no need for wavefunction renormalization of the superfields.

where powers of the renormalization mass μ have been introduced in order to deal with dimensionless renormalized couplings. In order to cancel the divergences in (8.2) we set

$$
\begin{aligned}
gZ_g &= \left[g - \frac{3\,\mathcal{N}}{2\,(4\pi)^2\,\epsilon}\,g^3\right] \equiv g + \frac{g_{(1)}}{\epsilon} \\
g_0 Z_{g_0} &= \left[g_0 + \frac{3\,\mathcal{N}}{2\,(4\pi)^2\,\epsilon}\,g_0^3\right] \equiv g_0 + \frac{g_{0(1)}}{\epsilon} \\
hZ_h &= \left[h - \frac{3\,\mathcal{N}}{2\,(4\pi)^2\,\epsilon}\,h^3\right] \equiv h + \frac{h_{(1)}}{\epsilon} \\
lZ_l &= \left[l - 3\frac{l\,g^2}{(4\pi)^2\,\epsilon}\left(\mathcal{N} + \frac{2l^2}{h^2\,\mathcal{N}} + \left(2 + \frac{g^2}{g_0^2}\right)\,\frac{2l^2}{\mathcal{N}\,(g_0^2 + g^2)}\right)\right] \equiv l + \frac{l_{(1)}}{\epsilon} \\
rZ_r &= r
\end{aligned}
\tag{9.2}
$$

We compute the β-functions by using the general prescription

$$
\beta_{\lambda_j} = -\epsilon\,\lambda_j - \lambda_{j(1)} + \sum_i \left(\lambda_i \frac{\partial \lambda_{j(1)}}{\lambda_i}\right) \tag{9.3}
$$

for any coupling λ_j. The β-functions for this theory turn out to be

$$
\begin{aligned}
\beta_g &= -\epsilon\,g - \frac{3\,\mathcal{N}}{(4\pi)^2}\,g^3 \\
\beta_{g_0} &= -\epsilon\,g_0 + \frac{3\,\mathcal{N}}{(4\pi)^2}\,g_0^3 \\
\beta_h &= -\epsilon\,h\ - \frac{3\,\mathcal{N}}{(4\pi)^2}\,h^3 \\
\beta_l &= -\epsilon\,l - 6\frac{l\,g^2}{(4\pi)^2}\left(\mathcal{N} + \frac{2l^2}{h^2\,\mathcal{N}} + \left(2 + \frac{g^2}{g_0^2}\right)\,\frac{2l^2}{\mathcal{N}\,(g_0^2 + g^2)}\right) \\
\beta_r &= -\epsilon\,r
\end{aligned}
\tag{9.4}
$$

We note that, despite the fact that the modified action (7.1) has an explicit dependence on the NAC parameter, there is no need to renormalize the constant two–form $\mathcal{F}^{\alpha\beta}$. Therefore, the star product does not get deformed by quantum corrections, consistently with what has been found at the component level [12].

It is manifest that the renormalization of r is trivial ($Z_r = 1$) since the term $\int d^4\theta\ \overline{\theta}^2 \mathrm{Tr}(\overline{\Gamma}^{\dot\alpha})\mathrm{Tr}(\overline{W}_{\dot\alpha})\mathrm{Tr}(\overline{W}^{\dot\beta}\ \overline{W}_{\dot\beta})$ does not receive any divergent correction. Therefore, in order to find the minimal renormalizable modification of the original deformation of the SYM theories, we can avoid introducing that term.

Moreover, from (9.2) it is easy to see that the renormalization functions of g, g_0 and h are equal up to a sign. This stems from the fact that vector loops do not contribute to the terms proportional to these coupling constants so that the ratio between the divergent contributions is fixed as in (6.4). In particular, this allows us to set $h^2 = -g_0^2$, or $h^2 = g^2$

keeping the renormalization procedure consistent at one–loop order. Therefore, the minimal number of coupling constants we need introduce to make the theory renormalizable at one–loop is *three* and the two minimal modified deformations of the $N = \frac{1}{2}$ SYM theory are

$$
\begin{aligned}
S_{min} = \; & \frac{1}{2\, g^2} \int d^4x\; d^4\theta \;\mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\; \overline{W}_{\dot\alpha}\right) \\
& + \frac{1}{2\, g_0^2\, \mathcal{N}} \int d^4x\; d^4\theta \Big[\;\mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\right) \mathrm{Tr}\left(\overline{W}_{\dot\alpha}\right) \\
& \qquad\qquad + 4i\mathcal{F}^{\rho\gamma}\; \overline{\theta}^2\; \mathrm{Tr}\left(\partial_{\rho\dot\rho}\overline{\Gamma}^{\dot\alpha}\right) \mathrm{Tr}\left(\overline{W}_{\dot\alpha}\overline{\Gamma}_{\gamma}{}^{\dot\rho}\right) \\
& \qquad\qquad - \mathcal{F}^2\; \overline{\theta}^2\; \mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\overline{W}_{\dot\alpha}\right) \mathrm{Tr}\left(\overline{W}^{\dot\beta}\overline{W}_{\dot\beta}\right) \;\Big] \\
& + \frac{1}{l^2}\, \mathcal{F}^2 \int d^4x\; d^4\theta\; \overline{\theta}^2\; \mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\; \overline{W}_{\dot\alpha}\; \overline{W}^{\dot\beta}\; \overline{W}_{\dot\beta}\right)
\end{aligned}
\tag{9.5}
$$

or

$$
\begin{aligned}
S'_{min} = \; & \frac{1}{2\, g^2} \int d^4x\; d^4\theta \Big[\;\mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\; \overline{W}_{\dot\alpha}\right) \\
& \qquad\qquad + \mathcal{F}^2\; \overline{\theta}^2\; \mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\overline{W}_{\dot\alpha}\right) \mathrm{Tr}\left(\overline{W}^{\dot\beta}\overline{W}_{\dot\beta}\right) \Big] \\
& + \frac{1}{2\, g_0^2\, \mathcal{N}} \int d^4x\; d^4\theta \Big[\;\mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\right) \mathrm{Tr}\left(\overline{W}_{\dot\alpha}\right) \\
& \qquad\qquad + 4i\mathcal{F}^{\rho\gamma}\; \overline{\theta}^2\; \mathrm{Tr}\left(\partial_{\rho\dot\rho}\overline{\Gamma}^{\dot\alpha}\right) \mathrm{Tr}\left(\overline{W}_{\dot\alpha}\overline{\Gamma}_{\gamma}{}^{\dot\rho}\right) \Big] \\
& + \frac{1}{l^2}\, \mathcal{F}^2 \int d^4\theta\; \overline{\theta}^2\; \mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\; \overline{W}_{\dot\alpha}\; \overline{W}^{\dot\beta}\; \overline{W}_{\dot\beta}\right)
\end{aligned}
\tag{9.6}
$$

One would be tempted to further reduce the number of independent couplings by setting $g_0^2 = -g^2$. However, this would lead to a dangerous cancellation between the quadratic terms of the $U(1)$ vector fields and, as it is evident from (D.11), this would make our procedure inconsistent.

10 Adding matter and going beyond one–loop

So far we have discussed the renormalization at one loop of a $N = \frac{1}{2}$ pure gauge theory classically described by the action (7.1). A natural extension of our analysis would follow two possible directions:

1) The inclusion of (anti)chiral matter and the evaluation of the corresponding effective action at one–loop;

2) The evaluation of higher order corrections to the effective action with or without matter.

We note that the evaluation of higher loop corrections to the gauge effective action necessarily involves lower order corrections to the ghost action. As explained in Section 3, ghosts formally behave as matter fields. Therefore, computing quantum corrections to the matter action gives direct informations about the effective action for ghosts and allows for higher order calculations in the gauge theory. The two lines of investigation are then strictly related.

In this Section we briefly address the main features of the calculation of the matter effective action without entering into any detail. We look for divergent corrections to the matter action

$$\int d^4\theta \; \Phi\overline{\Phi} \tag{10.1}$$

(we forget about the trace structure for the moment). We recall that Φ and $\overline{\Phi}$ are covariantly chiral and antichiral superfields; performing the quantum–background expansion of the action we obtain interaction vertices of the form chiral–antichiral–gauge. We can then look for one–loop corrections to these terms.

Following exactly the same arguments of Section 5, we select the general structure of matter–gauge corrections on the basis of dimensional arguments and R-symmetries, but neglecting for the moment gauge invariance considerations. For example we can look for terms of the form

$$\Lambda^\beta \int d^4x \; d^4\theta \; \overline{\theta}^{\overline{\tau}} \; \mathcal{F}^\alpha \; \Phi \; \overline{\Phi} \; D^\gamma \; \overline{D}^{\overline{\gamma}} \; \partial^\delta \; \overline{\Gamma}^{\overline{\sigma}} \tag{10.2}$$

where $0 \leq \overline{\tau} \leq 2$ and the dimensions and R-charges for the new objects are

	dim	R-charge
Φ	1	-1
$\overline{\Phi}$	1	1

In particular, the R-charges of the covariantly (anti)chiral fields have been fixed by requiring their masses to have vanishing R–charge.

Fixing the dimension and the R-charge of the generic counterterm (10.2) leads to the following relations

$$\alpha + \frac{\overline{\tau}}{2} - \frac{1}{2}\left(\gamma + \overline{\gamma} + \overline{\sigma}\right) - \delta \geq 0 \tag{10.3}$$

$$\overline{\sigma} + 2\alpha = \overline{\tau} + \gamma - \overline{\gamma} \tag{10.4}$$

from which it follows

$$\overline{\sigma} \leq \overline{\tau} - \delta - \overline{\gamma} \leq \overline{\tau} \leq 2 \tag{10.5}$$

As discussed in Section 5 this is supplemented by the extra constraint

$$\gamma \leq 2\overline{\sigma} \tag{10.6}$$

Other conditions come from the requirement of having all the spinorial indices correctly contracted, together with a nontrivial dependence on $\mathcal{F}^{\rho\gamma}$ (avoiding $\mathcal{F}^{\rho}{}_{\rho} = 0$, of course)

$$2\alpha + \delta + \gamma = 2n + 2 \qquad\qquad n \geq 1 \tag{10.7}$$

$$\overline{\tau} + \delta + \overline{g} + \overline{\sigma} = 2m \qquad\qquad m \geq 0 \tag{10.8}$$

where n, m are integers. Collecting all the constraints, it is easy to see that the only possible structures are

$$\mathcal{F}^{\rho\gamma} \int d^4x \; d^4\theta \; \overline{\theta}^2 \; \Phi \; \partial_{\rho}{}^{\dot{\alpha}}\overline{\Phi} \; \overline{\Gamma}_{\gamma\dot{\alpha}} \tag{10.9}$$

$$\mathcal{F}^{\rho\gamma} \int d^4x \; d^4\theta \; \overline{\theta}^2 \; \Phi \; \overline{\Phi} \; \partial_{\rho}{}^{\dot{\alpha}}\overline{\Gamma}_{\gamma\dot{\alpha}} \tag{10.10}$$

$$\mathcal{F}^{\rho\gamma} \int d^4x \; d^4\theta \; \overline{\theta}^2 \; \Phi \; \overline{\Phi} \; \overline{\Gamma}_{\gamma}{}^{\dot{\alpha}} \; \overline{\Gamma}_{\rho\dot{\alpha}} \tag{10.11}$$

$$\mathcal{F}^2 \int d^4x \; d^4\theta \; \overline{\theta}^2 \; \Phi \; \overline{\Phi} \; \overline{W}^{\dot{\alpha}} \; \overline{W}_{\dot{\alpha}} \tag{10.12}$$

Performing the reduction to components of each single term by using the conventions introduced in Appendix D, one can easily see the correspondence between these terms and the results obtained directly in components [12].

In superspace these contributions can arise from the sets of diagrams in Figures 6–9.

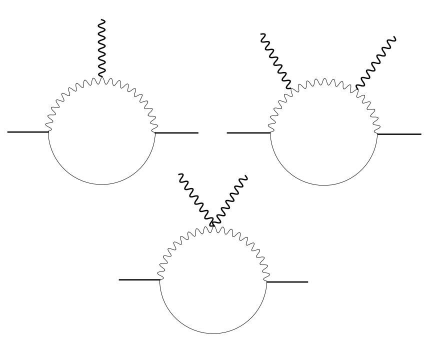

Figure 6

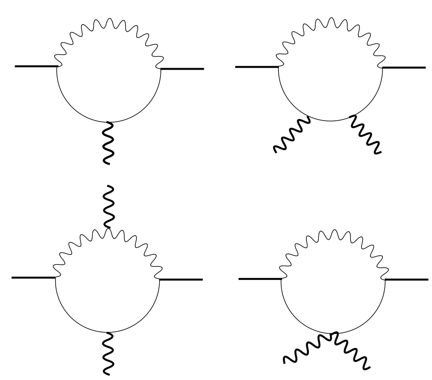

Figure 7

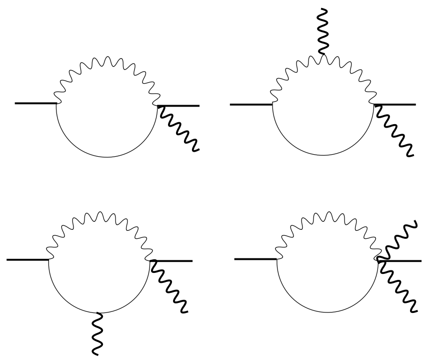

Figure 8

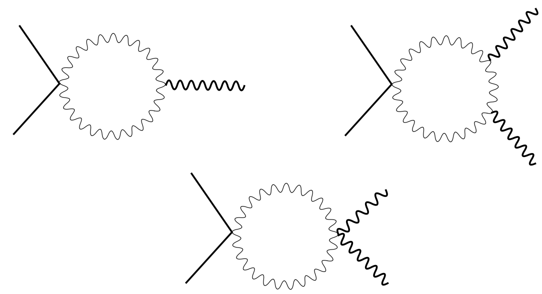

Figure 9

In Figs. 6 and 9 external gauge fields come from vertices arising from the expansion of the pure gauge part of the action (7.1). In Fig. 7 and 8 external vector legs inserted on a matter propagator and/or at a mixed vertex come from

$$\frac{\delta\Phi(z)}{\delta\Phi(z')} = \overline{\nabla}^2 \delta^{(8)}(z - z') \tag{10.13}$$

where we expand the covariant derivatives in powers of background gauge fields.

Without performing the complete evaluation of these contributions, we can give a straightforward argument to argue that some of them will be certainly present. The argument goes as follows: already in the ordinary anticommutative case, the matter propagator receives corrections from the diagram in Fig. 10

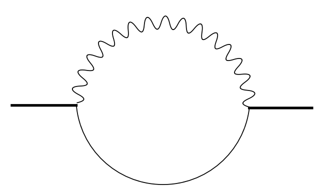

Figure 10

which gives rise to a divergent term of the form

$$\int d^4x\ d^4\theta\ \left[\mathcal{N}\mathrm{Tr}\left(\overline{\Phi}\ \Phi\right) - \mathrm{Tr}\left(\overline{\Phi}\right)\mathrm{Tr}\left(\Phi\right)\right] \tag{10.14}$$

This contribution is present also in the NAC case. In this case, however, the double trace piece is not supergauge invariant [21] and its supergauge variation is proportional to

$$\mathcal{F}^{\gamma\rho} \int d^4x\ d^4\theta\ \overline{\theta}^2\ \mathrm{Tr}\left(\partial_\rho{}^{\dot\rho}\overline{\Lambda}\ \partial_{\gamma\dot\rho}\overline{\Phi}\right)\ \mathrm{Tr}\left(\Phi\right) \tag{10.15}$$

Supergauge invariance of the effective action then requires extra divergent terms to emerge from diagrams 6–9 whose gauge variations compensate (10.15). In particular, the variation

(10.15) can be canceled by the variation of

$$\mathcal{F}^{\rho\gamma} \int d^4x \; d^4\theta \; \overline{\theta}^2 \; \mathrm{Tr}\left(\overline{\Gamma}_{\rho}^{\;\dot{\rho}} \; \partial_{\gamma\dot{\rho}}\overline{\Phi}\right) \; \mathrm{Tr}\,(\Phi) \tag{10.16}$$

Therefore, we expect such a a term to appear at one–loop order. This signals the necessity of modifying the classical matter action to make the theory renormalizable as we did for the pure gauge part. In components this has been already done in [12].

We now briefly discuss the evaluation of higher–loop contributions to the pure gauge effective action. In higher–loop diagrams, lower order corrections to the ghost propagator and vertices will be necessary. Therefore, in order to compute the gauge effective action we need go through the evaluation of the ghost effective action as well.

As already mentioned, ghosts behave formally as massless (anti)chiral matter, at least for what concerns Feynman rules. Therefore we can partially exploit the previous discussion about matter contributions. However, there is a crucial difference between matter and ghosts which should be clarified: while matter undergoes background–quantum splitting and one–loop divergent contributions to the matter background effective action require renormalization, ghosts are intrinsically quantum fields and do not appear in the background effective action. Divergent loops with external ghost lines (which can be obtained from the diagrams discussed above) appear only as subdiagrams in higher–order diagrams with external physical background lines. The corresponding subdivergences have to be BPHZ-subtracted out or compensated by introducing counterterms in the quantum action. Their form, and possible modifications due to the NAC effects would be controlled, or in fact determined by BRST invariance; but this is not the place for discussing these issues.

In the case of ghosts, the general argument given above to select all possible divergent structures which might appear in the ghost effective action should be modified since the R-charge assignment for ghosts is different. However, in this case we prefer to proceed by direct inspection and observe that the graphs contributing at one–loop are still the ones in Figs. 6–9 where now we have ghosts on the (anti)chiral lines. In particular, given the explicit form of the ghost action

$$\begin{aligned} S_{gh} \;\; = \;\; \int d^4x d^4\theta \Big[& \overline{c}'c - c'\overline{c} + \frac{1}{2}(c' + \overline{c}')[V,(c+\overline{c})] \\ & + \frac{1}{12}(c' + \overline{c}')[V,[V,(c-\overline{c})]] \Big] + \mathcal{O}(V^3) \end{aligned} \tag{10.17}$$

it is quite immediate to realize that graphs in Figs. 6,7,8 cancel due to an opposite sign in the $\langle \overline{c}'c \rangle$ and $\langle c'\overline{c} \rangle$ propagators. Therefore in the ghost case new structures, if any, will emerge only from diagrams in Fig. 9. These corrections have to be taken into account when we compute two loop corrections to the gauge effective action. We note that in the case of ghosts we cannot apply the argument which follows from eq. (10.14) to guarantee

that new structures will necessarily appear at the quantum level, as ghost self–energy corrections coming from diagrams as in Fig. 10 cancel [32].

As a final comment we consider the case of the $N = 4$ SYM theory in the presence of NAC geometry. In $N = 1$ superfield formalism, $N = 4$ SYM theory is described by a $N = 1$ gauge theory plus three chiral scalars in the adjoint representation, interacting with a cubic superpotential [32] (the deformed action in components is given in [33]). In this setup it is possible to discuss the quantum properties of the theory when ordinary products are replaced by $*$-products, so generically breaking $N = 4$ to $N = 1/2$. In particular we are interested in the renormalization of the gauge part. Looking at the result (6.4) it is evident that the gauge action (2.24) does not receive any divergent correction at one–loop since in this case $N_f = 3$. However, as already noticed, new matter–matter–gauge vertices of the form (10.9–10.12) can arise at one loop when we look at the matter part of the effective action. If these new terms appear, renormalization requires one inserting them already in the original action and, once quantized, they might spoil the cancellation of (6.4). Therefore, a complete analysis of one–loop contributions to the matter superfields has to be performed in order to make clear statements about the renormalizability of the gauge part of this theory. This issue is beyond the scope of the present work.

Acknowledgments

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A Mathematical tools

In this Appendix we list the main identities we used in the calculations concerning color traces and momentum loop integrals.

First of all we fix the group conventions for the $U(\mathcal{N})$ group. In the fundamental representation the generators are $\mathcal{N} \times \mathcal{N}$ unitary matrices T^A, $A = 0, \cdots, \mathcal{N}^2 - 1$, where $T^0 = \frac{\mathbb{1}}{\sqrt{\mathcal{N}}}$, whereas T^a are the $SU(\mathcal{N})$ generators. Their normalization is fixed by

$$\mathrm{Tr}(T^A T^B) = \delta^{AB} \tag{A.1}$$

The algebra of generators reads

$$[T^A, T^B] = i\ f^{ABC} T^C \tag{A.2}$$

where f^{ABC} are the structure constants given by

$$f^{abc} = -i \mathrm{Tr}(T^a [T^b, T^c]) \qquad , \qquad f^{0AB} = 0 \tag{A.3}$$

We also introduce

$$d^{abc} = \mathrm{Tr}(T^a \{T^b, T^c\}) \qquad , \qquad d^{0AB} = \frac{2}{\sqrt{N}} \delta^{AB} \tag{A.4}$$

Useful relations are:

$$\mathrm{Tr}\left(T^A T^B T^C\right) = \frac{1}{2}\left(i f^{ABC} + d^{ABC}\right) \tag{A.5}$$

$$\mathrm{Tr}\left(T^A T^B T^C T^D\right) = \frac{1}{4}\left(i f^{ABE} + d^{ABE}\right)\left(i f^{ECD} + d^{ECD}\right) \tag{A.6}$$

Given two scalar objects $M \equiv M^A T^A$ and $N \equiv N^A T^A$ in the adjoint representation of the gauge group, we have the general identity

$$\begin{aligned} [M, N]_* &= \frac{1}{2}\{T^A, T^B\}[M^A, N^B]_* \ + \ \frac{1}{2}[T^A, T^B]\{M^A, N^B\}_* \\ &= \frac{1}{2} d^{ABC}[M^A, N^B]_* T^C \ + \ \frac{i}{2} f^{ABC}\{M^A, N^B\}_* T^C \end{aligned} \tag{A.7}$$

The generalization of the above identity in the presence of spinorial quantities can be obtained straightforwardly.

We now consider momentum loop integrals. As stated in the main text, all the divergent contributions are expressed in terms of a self–energy integral $\mathcal{S}$ which in dimensional regularization ($n = 4 - 2\epsilon$) is

$$\mathcal{S} \equiv \int d^4 q\ \frac{1}{((q-p)^2 + m\overline{m})(q^2 + m\overline{m})} = \frac{1}{(4\pi)^2}\ \frac{1}{\epsilon} + \mathcal{O}(1) \tag{A.8}$$

Other one–loop divergent integrals are obtained in terms of $\mathcal{S}$ through the following identities

$$\int d^4q \ \frac{q_{\alpha\dot{\alpha}}}{((q-p)^2+m\overline{m})(q^2+m\overline{m})} = \frac{1}{2}p_{\alpha\dot{\alpha}} \ \mathcal{S} \tag{A.9}$$

$$\int d^4q \frac{q_{\alpha\dot{\alpha}}q_{\beta\dot{\beta}}}{(q^2+m\overline{m})((q+p)^2+m\overline{m})((q+r)^2+m\overline{m})} \sim \frac{1}{2}C_{\alpha\beta}C_{\dot{\alpha}\dot{\beta}} \ \mathcal{S} \tag{A.10}$$

$$\int d^4q \frac{q_{\alpha\dot{\alpha}}q_{\beta\dot{\beta}}q_{\gamma\dot{\gamma}}q_{\rho\dot{\rho}}}{(q^2+m\overline{m})((q+p)^2+m\overline{m})((q+r)^2+m\overline{m})((q+s)^2+m\overline{m})} \sim$$
$$\frac{1}{6}(C_{\alpha\beta}C_{\dot{\alpha}\dot{\beta}}C_{\gamma\rho}C_{\dot{\gamma}\dot{\rho}} + C_{\alpha\gamma}C_{\dot{\alpha}\dot{\gamma}}C_{\beta\rho}C_{\dot{\beta}\dot{\rho}} + C_{\alpha\rho}C_{\dot{\alpha}\dot{\rho}}C_{\beta\gamma}C_{\dot{\beta}\dot{\gamma}}) \ \mathcal{S} \tag{A.11}$$

Since the divergent part of these integrals is mass independent, the same results hold also for $m = \overline{m} = 0$ (ghost contributions).

B Momentum superspace

In this Appendix we give a detailed description of all the mathematical tools for Fourier transformed superspace.

We first consider the case of ordinary, anticommuting superspace. We define a *momentum superspace* described by coordinates $(p_{\alpha\dot{\alpha}}, \pi_\alpha, \overline{\pi}_{\dot{\alpha}})$ conjugate to $(x^{\alpha\dot{\alpha}}, \theta^\alpha, \overline{\theta}^{\dot{\alpha}})$. The derivatives with respect to the spinorial momenta

$$\frac{\partial}{\partial\pi^\alpha} \equiv \eth_\alpha \qquad\qquad \frac{\partial}{\partial\overline{\pi}^{\dot{\alpha}}} \equiv \overline{\eth}_{\dot{\alpha}} \tag{B.1}$$

satisfy $\eth_\alpha \pi^\beta = \delta_\alpha{}^\beta$, $\eth^\alpha \pi_\beta = -\delta_\beta{}^\alpha$. In analogy with the case of ordinary superspace, we define the integration as

$$\int d^2\pi \ \Phi(\pi) = \eth^2\Phi| \qquad , \qquad \int d^2\overline{\pi} \ \Phi(\overline{\pi}) = \overline{\eth}^2\Phi| \tag{B.2}$$

In particular, this implies

$$\int d^2\pi \ \pi^2 \equiv \eth^2\pi^2 = -1 \qquad , \qquad \int d^2\overline{\pi} \ \overline{\pi}^2 \equiv \overline{\eth}^2\overline{\pi}^2 = -1 \tag{B.3}$$

so that, consistently, the momentum delta functions are given by $\delta^{(2)}(\pi) \equiv -\pi^2$, $\delta^{(2)}(\overline{\pi}) \equiv -\overline{\pi}^2$.

We define the Fourier transform (FT) in the bosonic coordinates and its inverse as

$$\widetilde{\Phi}(p) = \int d^4x \ e^{ipx} \ \Phi(x) \qquad , \qquad \Phi(x) = \int d^4p \ e^{-ipx} \ \widetilde{\Phi}(p) \tag{B.4}$$

(omitting $(2\pi)^4$ factors).

Analogously, the FT on the fermionic coordinates and its inverse are given by

$$\widetilde{\Phi}(\pi,\overline{\pi}) = \int d^2\theta d^2\overline{\theta}\; e^{i\pi\theta + i\overline{\pi}\overline{\theta}}\;\Phi(\theta,\overline{\theta}) \qquad , \qquad \Phi(\theta,\overline{\theta}) = \int d^2\pi d^2\overline{\pi}\; e^{-i\pi\theta - i\overline{\pi}\overline{\theta}}\;\widetilde{\Phi}(\pi,\overline{\pi}) \quad \text{(B.5)}$$

The consistency of the two expressions in (B.5) follows from the identities

$$\begin{aligned} &\int d^2\pi d^2\overline{\pi}\; e^{i\pi(\theta-\theta') + i\overline{\pi}(\overline{\theta}-\overline{\theta}')} = \delta^{(4)}(\theta-\theta') \\ &\int d^2\theta d^2\overline{\theta}\; e^{i(\pi-\pi')\theta + i(\overline{\pi}-\overline{\pi}')\overline{\theta}'} = \delta^{(4)}(\pi-\pi') \end{aligned} \qquad \text{(B.6)}$$

where, as usual, $\delta^{(2)}(\theta-\theta') \equiv -(\theta-\theta')^2$.

From the definitions given above we have the following identifications between conjugate variables

$$\begin{aligned} p_{\alpha\dot{\alpha}} &= i\partial_{\alpha\dot{\alpha}} \qquad , \qquad x_{\alpha\dot{\alpha}} = -i\frac{\partial}{\partial p^{\alpha\dot{\alpha}}} \\ \pi_\alpha &= i\partial_\alpha \qquad\quad , \qquad \theta_\alpha = -i\eth_\alpha \\ \overline{\pi}_{\dot{\alpha}} &= i\overline{\partial}_{\dot{\alpha}} \qquad\quad , \qquad \overline{\theta}_{\dot{\alpha}} = -i\overline{\eth}_{\dot{\alpha}} \end{aligned} \qquad \text{(B.7)}$$

B.1 Covariant derivatives

We use chiral representation for the superspace covariant derivatives $D_\alpha, \overline{D}_{\dot{\alpha}}$ [26]. Performing FT, in momentum superspace we obtain momentum operators $\widetilde{D}_\alpha, \widetilde{\overline{D}}_{\dot{\alpha}}$ given by (see identities (B.7))

$$\begin{aligned} D_\alpha = \partial_\alpha + i\overline{\theta}^{\dot{\alpha}}\partial_{\alpha\dot{\alpha}} \qquad &\rightarrow \qquad \widetilde{D}_\alpha = -i\pi_\alpha - i\overline{\eth}^{\dot{\alpha}} p_{\alpha\dot{\alpha}} \\ \overline{D}_{\dot{\alpha}} = \overline{\partial}_{\dot{\alpha}} \qquad\qquad\quad &\rightarrow \qquad \widetilde{\overline{D}}_{\dot{\alpha}} = -i\overline{\pi}_{\dot{\alpha}} \end{aligned} \qquad \text{(B.8)}$$

and

$$\begin{aligned} \widetilde{D}^2 &= -\pi^2 - \pi^\alpha\overline{\eth}^{\dot{\alpha}} p_{\alpha\dot{\alpha}} - \overline{\eth}^2 p^2 \\ \widetilde{\overline{D}}^2 &= -\overline{\pi}^2 \end{aligned} \qquad \text{(B.9)}$$

(Anti)commutation rules for $\widetilde{D}$–derivatives [††] are

$$\{\widetilde{D}_\alpha, \widetilde{\overline{D}}_{\dot{\alpha}}\} = p_{\alpha\dot{\alpha}} \qquad\qquad \text{the rest } = 0 \qquad \text{(B.10)}$$

[††]With abuse of language we call them “derivatives” even if some are actually multiplicative operators.

and the following identities hold

$$
\begin{aligned}
&[\widetilde{D}^\alpha, \widetilde{\overline{D}}^2] = p^{\alpha\dot{\alpha}}\widetilde{\overline{D}}_{\dot{\alpha}} && [\widetilde{\overline{D}}^{\dot{\alpha}}, \widetilde{D}^2] = p^{\alpha\dot{\alpha}}\widetilde{D}_\alpha \\
&\widetilde{D}^2\widetilde{\overline{D}}^2\widetilde{D}^2 = -p^2\widetilde{D}^2 && \widetilde{\overline{D}}^2\widetilde{D}^2\widetilde{\overline{D}}^2 = -p^2\widetilde{\overline{D}}^2 \\
&-p^2 = \widetilde{\overline{D}}^2\widetilde{D}^2 + \widetilde{D}^2\widetilde{\overline{D}}^2 - \widetilde{\overline{D}}^{\dot{\alpha}}\widetilde{D}^2\widetilde{\overline{D}}_{\dot{\alpha}} = \widetilde{\overline{D}}^2\widetilde{D}^2 + \widetilde{D}^2\widetilde{\overline{D}}^2 - \widetilde{D}^\alpha\widetilde{\overline{D}}^2\widetilde{D}_\alpha
\end{aligned}
\tag{B.11}
$$

Momentum covariant derivatives can be integrated by parts according to the following rule

$$
\int d^4p\; d^4\pi \widetilde{D}_\alpha(p,\pi)\widetilde{\Phi}(p,\pi)\widetilde{\Psi}(-p,-\pi) \;=\; -\int d^4p\; d^4\pi \widetilde{\Phi}(p,\pi)\widetilde{D}_\alpha(-p,-\pi)\widetilde{\Psi}(-p,-\pi)
\tag{B.12}
$$

B.2 (Anti)Chirality conditions

Given the superfield $V(x,\theta,\overline{\theta})$ expanded in powers of spinorial coordinates, it is easy to determine its expansion in momentum superspace by Fourier transforming term by term through the following identities

$$
\begin{aligned}
&\int d^2\theta d^2\overline{\theta}\; e^{i\pi\theta + i\overline{\pi}\overline{\theta}}\; 1 = \pi^2\overline{\pi}^2 = \delta^{(4)}(\pi) \\
&\int d^2\theta d^2\overline{\theta}\; e^{i\pi\theta + i\overline{\pi}\overline{\theta}}\; \theta^\alpha = -i\pi^\alpha\overline{\pi}^2 = i\pi^\alpha\delta^{(2)}(\overline{\pi}) \\
&\int d^2\theta d^2\overline{\theta}\; e^{i\pi\theta + i\overline{\pi}\overline{\theta}}\; \theta^2 = \overline{\pi}^2 = -\delta^{(2)}(\overline{\pi})
\end{aligned}
\tag{B.13}
$$

and analogous ones for the antichiral sector. It is easy to see that the expansion

$$
\begin{aligned}
V(x,\theta,\overline{\theta}) = C(x) + \theta^\alpha\chi_\alpha(x) + \overline{\theta}^{\dot{\alpha}}\overline{\chi}_{\dot{\alpha}}(x) - \theta^2 M(x) - \overline{\theta}^2\overline{M}(x) \\
+ \theta^\alpha\overline{\theta}^{\dot{\alpha}}A_{\alpha\dot{\alpha}}(x) - \overline{\theta}^2\theta^\alpha\lambda_\alpha(x) - \theta^2\overline{\theta}^{\dot{\alpha}}\overline{\lambda}_{\dot{\alpha}}(x) + \theta^2\overline{\theta}^2 D'(x)
\end{aligned}
\tag{B.14}
$$

in momentum superspace corresponds to

$$
\begin{aligned}
\widetilde{V}(p,\pi,\overline{\pi}) = \pi^2\overline{\pi}^2\widetilde{C}(p) - i\overline{\pi}^2\pi^\alpha\widetilde{\chi}_\alpha(p) - i\pi^2\overline{\pi}^{\dot{\alpha}}\widetilde{\overline{\chi}}_{\dot{\alpha}}(p) - \overline{\pi}^2\widetilde{M}(p) - \pi^2\widetilde{\overline{M}}(p) \\
- \pi^\alpha\overline{\pi}^{\dot{\alpha}}\widetilde{A}_{\alpha\dot{\alpha}}(p) - i\pi^\alpha\widetilde{\lambda}_\alpha(p) - i\overline{\pi}^{\dot{\alpha}}\widetilde{\overline{\lambda}}_{\dot{\alpha}}(p) + \widetilde{D'}(p)
\end{aligned}
\tag{B.15}
$$

We now discuss constrained superfields. Going to momentum superspace the chiral constraint on a superfield becomes

$$
\overline{D}_{\dot{\alpha}}\Phi(\theta,\overline{\theta}) = 0 \qquad \to \qquad \widetilde{\overline{D}}_{\dot{\alpha}}\widetilde{\Phi}(\pi,\overline{\pi}) = (-i\overline{\pi}_{\dot{\alpha}})\widetilde{\Phi}(\pi,\overline{\pi}) = 0
\tag{B.16}
$$

The solution to this constraint is necessarily of the form $\widetilde{\Phi}(\pi,\overline{\pi}) = \overline{\pi}^2\chi(\pi)$, where χ is an unconstrained superfield independent of $\overline{\pi}$. The solution can be also written as

$$\widetilde{\Phi}(\pi,\overline{\pi}) = \overline{\pi}^2 \left(\overline{\eth}^2\widetilde{\Phi}(\pi,\overline{\pi})|_{\overline{\pi}=0}\right) = -\delta^{(2)}(\overline{\pi})\,\overline{\eth}^2\widetilde{\Phi}(\pi,\overline{\pi}) \tag{B.17}$$

Going to components the most general expression for a chiral superfield is

$$\Phi(x,\theta,\overline{\theta}) = \phi(y) + \theta^\alpha\psi_\alpha(y) - \theta^2 F(y) \tag{B.18}$$

where $y^{\alpha\dot{\alpha}} = x^{\alpha\dot{\alpha}} - i\theta^\alpha\overline{\theta}^{\dot{\alpha}}$. If we perform the FT keeping y as an independent variable we obtain (see eqs. (B.13))

$$\widetilde{\Phi} = -\overline{\pi}^2(F + i\pi^\alpha\psi_\alpha - \pi^2\phi) = \delta^{(2)}(\overline{\pi})(F + i\pi^\alpha\psi_\alpha - \pi^2\phi) \tag{B.19}$$

A useful identity satisfied by a chiral superfield is

$$\widetilde{\overline{D}}^2\overline{\eth}^2\widetilde{\Phi} = \widetilde{\Phi} \tag{B.20}$$

For an antichiral superfield the constraint becomes

$$D_\alpha\overline{\Phi}(\theta,\overline{\theta}) = 0 \qquad \rightarrow \qquad \widetilde{D}_\alpha\widetilde{\overline{\Phi}}(\pi,\overline{\pi}) = (-i\pi_\alpha - ip_{\alpha\dot{\alpha}}\overline{\eth}^{\dot{\alpha}})\widetilde{\overline{\Phi}}(\pi,\overline{\pi}) = 0 \tag{B.21}$$

This is equivalent to $\overline{\eth}_{\dot{\alpha}}\widetilde{\overline{\Phi}} = -\frac{p_{\alpha\dot{\alpha}}}{p^2}\pi^\alpha\widetilde{\overline{\Phi}}$ which implies

$$\overline{\eth}_{\dot{\alpha}}\widetilde{\overline{\Phi}}|_{\pi=0} = 0 \tag{B.22}$$

For the antichiral superfield the $(\pi,\overline{\pi})$-expansion is more complicated. If we introduce $\overline{y}^{\alpha\dot{\alpha}} \equiv y^{\alpha\dot{\alpha}} - i\theta^\alpha\overline{\theta}^{\dot{\alpha}}$, $D_\beta\overline{y}^{\alpha\dot{\alpha}} = 0$, the expansion of an antichiral superfield in coordinate superspace

$$\overline{\Phi}(\overline{y},\theta,\overline{\theta}) = \overline{\phi}(\overline{y}) + \overline{\theta}^{\dot{\alpha}}\overline{\psi}_{\dot{\alpha}}(\overline{y}) - \overline{\theta}^2\overline{F}(\overline{y}) \tag{B.23}$$

when transformed to momentum superspace becomes

$$\widetilde{\overline{\Phi}}(p,\pi,\overline{\pi}) = \widetilde{\overline{\phi}}(p)(\pi^2\overline{\pi}^2 + \pi^\alpha\overline{\pi}^{\dot{\alpha}}p_{\alpha\dot{\alpha}} - p^2) - i(\pi^2\overline{\pi}^{\dot{\alpha}} + \pi_\alpha p^{\alpha\dot{\alpha}})\widetilde{\overline{\psi}}_{\dot{\alpha}}(p) - \pi^2\widetilde{\overline{F}}(p) \tag{B.24}$$

The functional derivatives with respect to a (anti)chiral superfield in momentum superspace are defined as

$$\begin{aligned}
\frac{\delta\widetilde{\Phi}(\pi_1)}{\delta\widetilde{\Phi}(\pi_2)} &= \widetilde{\overline{D}}^2\delta^{(4)}(\pi_1 - \pi_2) \\
\frac{\delta\widetilde{\overline{\Phi}}(\pi_1)}{\delta\widetilde{\overline{\Phi}}(\pi_2)} &= \widetilde{D}^2\delta^{(4)}(\pi_1 - \pi_2)
\end{aligned} \tag{B.25}$$

This can be easily understood by observing that the condition

$$\frac{\delta}{\delta\Phi_1(\theta)}\int d^2\theta'\Phi_1(\theta')\Phi_2(\theta') \;=\; \Phi_2(\theta) \tag{B.26}$$

is translated in momentum superspace as

$$\frac{\delta}{\delta\widetilde{\Phi}_1(\pi)}\int d^4\pi'\widetilde{\Phi}_1(\pi')\frac{\widetilde{D}^2}{\Box}\widetilde{\Phi}_2(-\pi') \;=\; \widetilde{\Phi}_2(-\pi) \tag{B.27}$$

This equation is satisfied by (B.25).

B.3 The star product in momentum superspace

We now extend the previous definitions to $N=\frac{1}{2}$ superspace. This requires rotating to euclidean space and turning on the nontrivial anticommutators (2.1). Since the derivatives $\frac{\partial}{\partial\theta^\alpha}$ do not get affected by nonanticommutativity, the spinorial variables $(\pi,\overline{\pi})$ in momentum superspace remain anticommuting.

As in the NC bosonic case, in momentum superspace the effects of noncommutativity are visible in the product of fields through the appearance of momentum phase factors. We apply FT (B.5) to the NC product of two superfields as given in (2.4). We begin by noting that

$$\left(e^{i\pi\theta}\right)_* = \left(e^{i\pi\theta}\right) \tag{B.28}$$

as a consequence of the fact that the new terms generated by the $*$-product are all proportional to the structure $\pi_\alpha\mathcal{F}^{\alpha\beta}\pi_\beta$ which vanishes, since $\mathcal{F}$ symmetric. Moreover, the following identity holds

$$e^{-i\pi\theta} * e^{-i\pi'\theta} = e^{-i(\pi+\pi')\theta}e^{\pi\wedge\pi'} \tag{B.29}$$

Therefore, when computing the NC product of two superfields in momentum superspace we find

$$\begin{aligned}\Phi(\theta,\overline{\theta}) * \Psi(\theta,\overline{\theta}) &= \int d^4\pi d^4\pi'\left(e^{-i\pi\theta-i\overline{\pi}\overline{\theta}}\;\widetilde{\Phi}(\pi,\overline{\pi})\right) * \left(e^{-i\pi'\theta-i\overline{\pi}'\overline{\theta}}\;\widetilde{\Psi}(\pi',\overline{\pi}')\right)\\ &= \int d^4\pi d^4\pi' e^{\pi\wedge\pi'}e^{-i(\pi+\pi')\theta}e^{-i(\overline{\pi}+\overline{\pi}')\overline{\theta}}\;\widetilde{\Phi}(\pi,\overline{\pi})\widetilde{\Psi}(\pi',\overline{\pi}')\end{aligned} \tag{B.30}$$

Thus, in momentum superspace the star product manifests itself through the phase factor $e^{\pi\wedge\pi'}$.

C One–loop non–renormalizability in components

In this Appendix we perform the reduction to components of our one–loop result in (6.4) and prove at component level the non–renormalizability of the original action (3.4).

We work in the Wess–Zumino gauge defined by the usual conditions $V| = DV| = \overline{D}V| = D^2V| = \overline{D}^2V| = 0$. While we use the ordinary definitions for the following component fields

$$
\begin{aligned}
f_{\alpha\beta} &= -\frac{1}{2}\partial_{\alpha}{}^{\dot\alpha} A_{\beta\dot\alpha} - \frac{1}{2}\partial_{\beta}{}^{\dot\alpha} A_{\alpha\dot\alpha} + \frac{i}{2}[A_{\alpha}{}^{\dot\alpha}, A_{\beta\dot\alpha}] \\
\overline{f}_{\dot\alpha\dot\beta} &= -\frac{1}{2}\partial_{\dot\alpha}{}^{\alpha} A_{\alpha\dot\alpha} - \frac{1}{2}\partial^{\alpha}{}_{\dot\beta} A_{\alpha\dot\alpha} + \frac{i}{2}[A^{\alpha}{}_{\dot\alpha}, A_{\alpha\dot\beta}] \\
D^2\overline{D}_{\dot\alpha}V\Big| &= \overline{\lambda}_{\dot\alpha} \qquad \overline{D}_{\dot\beta}D^2\overline{D}_{\dot\alpha}V\Big| = -iC_{\dot\alpha\dot\beta}D' + \overline{f}_{\dot\alpha\dot\beta}
\end{aligned}
\tag{C.1}
$$

we perform the shift [2]

$$
i\overline{D}^2 D_{\alpha}V\Big| = \lambda_{\alpha} + \vartheta_{\alpha} \qquad\qquad \vartheta_{\alpha} = -\frac{1}{2}\mathcal{F}_{\alpha}{}^{\rho}\{\overline{\lambda}^{\dot\beta}, A_{\rho\dot\beta}\}
\tag{C.2}
$$

in order to simplify the expansions. With these conventions we have

$$
\begin{aligned}
\overline{W}^{\dot\alpha}\Big| &= \overline{\lambda}^{\dot\alpha} \\
\overline{D}^{\dot\beta}\overline{W}^{\dot\alpha}\Big| &= -iC^{\dot\alpha\dot\beta}D' + \overline{f}^{\dot\alpha\dot\beta} \\
\overline{D}^2\overline{W}^{\dot\alpha}\Big| &= i\partial^{\alpha\dot\alpha}(\lambda_{\alpha} + \vartheta_{\alpha}) + [A^{\alpha\dot\alpha}, \lambda_{\alpha} + \vartheta_{\alpha}] \\
&\quad + \mathcal{F}^{\gamma\rho}\Big[\frac{1}{2}\{\overline{\lambda}^{\dot\alpha}, i\partial_{\rho}{}^{\dot\rho}A_{\gamma\dot\rho}\} + \frac{1}{2}\{\overline{\lambda}_{\dot\rho}, i\partial_{\rho}{}^{\dot\rho}A_{\gamma}{}^{\dot\alpha}\} + \frac{1}{2}\{i\partial_{\gamma}{}^{\dot\rho}\overline{\lambda}^{\dot\alpha}, A_{\rho\dot\rho}\} + \frac{1}{2}\{i\partial_{\rho}{}^{\dot\rho}\overline{\lambda}_{\dot\rho}, A_{\gamma}{}^{\dot\alpha}\} \\
&\qquad + \frac{1}{2}\overline{\lambda}^{\dot\beta}A_{\gamma}{}^{\dot\alpha}A_{\rho\dot\beta} + \frac{1}{2}A_{\gamma}{}^{\dot\beta}\overline{\lambda}^{\dot\alpha}A_{\rho\dot\beta} + \frac{1}{2}A_{\gamma}{}^{\dot\beta}A_{\rho}{}^{\dot\alpha}\overline{\lambda}_{\dot\beta}\Big] \\
&\quad + \mathcal{F}^2\Big[\frac{1}{4}\overline{\lambda}^{\dot\beta}\overline{\lambda}^{\dot\alpha}\overline{\lambda}_{\dot\beta}\Big]
\end{aligned}
\tag{C.3}
$$

Useful identities are

$$
\begin{aligned}
& i\partial^{\alpha\dot\alpha}\vartheta_{\alpha} = \frac{1}{2}\mathcal{F}^{\gamma\rho}\partial_{\rho\dot\rho}\Big[\{\overline{\lambda}^{\dot\rho}, A_{\gamma}{}^{\dot\alpha}\} - \{\overline{\lambda}^{\dot\alpha}, A_{\gamma}{}^{\dot\rho}\}\Big] \\
& [A^{\alpha\dot\alpha}, \vartheta_{\alpha}] - \frac{1}{2}\mathcal{F}^{\gamma\rho}[\overline{\lambda}_{\dot\beta}A_{\gamma}{}^{\dot\alpha}A_{\rho}{}^{\dot\beta} + A_{\gamma\dot\beta}\overline{\lambda}^{\dot\alpha}A_{\rho}{}^{\dot\beta} + A_{\gamma\dot\beta}A_{\rho}{}^{\dot\alpha}\overline{\lambda}^{\dot\beta}] = \\
& \qquad \frac{1}{2}\mathcal{F}^{\gamma\rho}[2A_{\gamma}{}^{\dot\beta}\overline{\lambda}^{\dot\alpha}A_{\rho\dot\beta} + \overline{\lambda}^{\dot\alpha}A_{\gamma}{}^{\dot\beta}A_{\rho\dot\beta} + A_{\gamma}{}^{\dot\beta}A_{\rho\dot\beta}\overline{\lambda}^{\dot\alpha}]
\end{aligned}
\tag{C.4}
$$

Using the property Tr $(\overline{\lambda}^{\dot\beta}\overline{\lambda}^{\dot\alpha}\overline{\lambda}_{\dot\beta}) = 0$, the classical gauge action (2.24) in components reads

$$
\begin{aligned}
S_{inv} \;=\; \frac{1}{g^2}\int d^4x\, \Big\{ & \mathrm{Tr}(D'^{\,2} - \frac{1}{2}\overline{f}^{\dot\alpha\dot\beta}\overline{f}_{\dot\alpha\dot\beta} + i\partial^{\alpha\dot\alpha}\overline{\lambda}_{\dot\alpha}\lambda_{\alpha} + \Big[A^{\alpha\dot\alpha}, \overline{\lambda}_{\dot\alpha}\Big]\lambda_{\alpha}) \\
& -i\mathcal{F}^{\gamma\rho}\ \mathrm{Tr}(f_{\gamma\rho}\overline{\lambda}^{\dot\alpha}\overline{\lambda}_{\dot\alpha}) + \frac{1}{2}\ \mathcal{F}^2\ \mathrm{Tr}(\overline{\lambda}^{\dot\alpha}\overline{\lambda}_{\dot\alpha}\overline{\lambda}^{\dot\beta}\overline{\lambda}_{\dot\beta})\Big\}
\end{aligned}
\tag{C.5}
$$

Similarly, we perform the reduction of the one–loop divergent contributions (6.4) obtaining

$$\Gamma^{(1)\ comp}_{gauge} = \frac{(-3+N_f)}{(4\pi)^2\epsilon} \int d^4x \Big\{ \mathcal{N}\ \mathrm{Tr}\Big(D'^{\ 2} - \frac{1}{2}\overline{f}^{\dot\alpha\dot\beta}\overline{f}_{\dot\alpha\dot\beta} + i\partial^{\alpha\dot\alpha}\overline{\lambda}_{\dot\alpha}\lambda_\alpha + [A^{\alpha\dot\alpha}, \overline{\lambda}_{\dot\alpha}]\lambda_\alpha \Big)$$
$$- \Big(\mathrm{Tr}(D')\mathrm{Tr}(D') - \frac{1}{2}\mathrm{Tr}(\overline{f}^{\dot\alpha\dot\beta})\mathrm{Tr}(\overline{f}_{\dot\alpha\dot\beta}) + i\partial^{\alpha\dot\alpha}\mathrm{Tr}(\overline{\lambda}_{\dot\alpha})\mathrm{Tr}(\lambda_\alpha) \Big)$$
$$- i\mathcal{N}\mathcal{F}^{\gamma\rho}\ \mathrm{Tr}(f_{\gamma\rho}\overline{\lambda}^{\dot\alpha}\overline{\lambda}_{\dot\alpha}) + i\ \mathcal{F}^{\gamma\rho}\ \mathrm{Tr}(f_{\gamma\rho})\mathrm{Tr}(\overline{\lambda}^{\dot\alpha}\overline{\lambda}_{\dot\alpha})$$
$$+ \frac{1}{2}\ \mathcal{F}^2\ \mathcal{N}\ \mathrm{Tr}(\overline{\lambda}^{\dot\alpha}\overline{\lambda}_{\dot\alpha}\overline{\lambda}^{\dot\beta}\overline{\lambda}_{\dot\beta}) - \frac{1}{2}\ \mathcal{F}^2\ \mathrm{Tr}(\overline{\lambda}^{\dot\alpha}\overline{\lambda}_{\dot\alpha})\mathrm{Tr}(\overline{\lambda}^{\dot\beta}\overline{\lambda}_{\dot\beta}) \Big\} \tag{C.6}$$

In order to renormalize the theory it is convenient to perform the following rescaling on the fields

$$A,\ \lambda,\ \overline{\lambda},\ D' \qquad \rightarrow \qquad g\ A,\ g\ \lambda,\ g\ \overline{\lambda},\ g\ D' \tag{C.7}$$

so that cubic terms in the action are proportional to g, quartic terms to g^2, while kinetic terms are independent of the coupling constant and will undergo wave function renormalization.

Since the one–loop divergent contributions independent of the NAC parameter are the same as for the ordinary $N=1$ SYM, the renormalization functions are fixed by

$$Z_{A_a} = Z^{\frac{1}{2}}_{\lambda_a} \cdot Z^{\frac{1}{2}}_{\overline{\lambda}_a} = Z_{D'_a} = Z_g^{-2} = 1 - \frac{(N_f-3)}{(4\pi)^2\epsilon}\mathcal{N}g^2$$
$$Z_{A_0} = Z_{\lambda_0} = Z_{\overline{\lambda}_0} = Z_{D'_0} = 1 \tag{C.8}$$

where for any field we have defined $\Phi = Z^{-\frac{1}{2}}\Phi_B$ and $g = Z_g^{-1}g_B$ for the coupling constant. Note that in the ordinary case one is forced to choose $Z_{\lambda_a} = Z_{\overline{\lambda}_a}$, whereas in the present case they could be different, as we are working in euclidean space where the two fermions are not related by h.c. conditions.

Armed with the renormalization functions (C.8) we then study the new contributions proportional to the NAC parameter. We concentrate on the term $(\mathcal{F}^{\gamma\rho}\ f_{\gamma\rho}\overline{\lambda}^2)$ which we rewrite as

$$\mathrm{Tr}(f_{\gamma\rho}\overline{\lambda}^{\dot\alpha}\overline{\lambda}_{\dot\alpha}) = C^{aBC} f^a_{\gamma\rho}\ \overline{\lambda}^{\dot\alpha B}\overline{\lambda}^C_{\dot\alpha} + \mathcal{N}^{-\frac{1}{2}} f^0_{\gamma\rho}\ \overline{\lambda}^{\dot\alpha B}\overline{\lambda}^B_{\dot\alpha} \tag{C.9}$$

where $C^{aBC} = \frac{1}{2}(i\ f^{aBC} + d^{aBC})$.

In the one–loop result (C.6) it is easy to see that the term proportional to the U(1) field strength actually cancels, whereas the divergent contribution to the term proportional to the nonabelian $f^a_{\gamma\rho}$ reads

$$-\frac{(N_f-3)}{(4\pi)^2\epsilon}\mathcal{N}g^2\ \mathcal{F}^{\gamma\rho} \int d^4x\ C^{aBC}\ i f^a_{\gamma\rho}\ \overline{\lambda}^{\dot\alpha B}\ \overline{\lambda}^C_{\dot\alpha} \tag{C.10}$$

Therefore the terms in the classical action (C.5)

$$g\ \mathcal{F}^{\gamma\rho}\ f^0_{\gamma\rho}\ \overline{\lambda}^{\dot\alpha\ B}\ \overline{\lambda}^B_{\dot\alpha} = g\ \mathcal{F}^{\gamma\rho}\ f^0_{\gamma\rho}\ \overline{\lambda}^{\dot\alpha\ 0}\ \overline{\lambda}^0_{\dot\alpha} + g\ \mathcal{F}^{\gamma\rho}\ f^0_{\gamma\rho}\ \overline{\lambda}^{\dot\alpha\ b}\ \overline{\lambda}^b_{\dot\alpha} \tag{C.11}$$

do not renormalize. This is made possible if separately the two terms in (C.11) do not renormalize, i.e. if

$$\begin{aligned} Z_{\mathcal{F}} Z_g &= 1 \\ Z_{\mathcal{F}} Z_g \left(Z^{\frac{1}{2}}_{\overline{\lambda}_b} \right)^2 &= 1 \end{aligned} \tag{C.12}$$

hold separately. Since in the NAC case we can choose λ and $\overline{\lambda}$ (and their renormalization functions) to be independent, the previous conditions have a solution compatible with (C.8). Precisely, we choose

$$\begin{aligned} &Z_{\overline{\lambda}_a} = 1 \\ &Z^{\frac{1}{2}}_{\lambda_a} = Z_{A_a} \\ &Z_{\mathcal{F}} = Z_g^{-1} = Z^{\frac{1}{2}}_{A_a} \end{aligned} \tag{C.13}$$

At this point all the renormalization functions have been fixed and no freedom is left. What we need check is whether the choice (C.13) is good enough to cancel the rest of the divergences, i.e the ones proportional to $g\ \mathcal{F}^{\gamma\rho}\ C^{aBC}\ f^a_{\gamma\rho}\ \overline{\lambda}^{\dot\alpha\ B}\ \overline{\lambda}^C_{\dot\alpha}$ and the ones proportional to the product of four $\overline{\lambda}$. We analyze them separately.

a) Summing all the contributions proportional to $g\ \mathcal{F}^{\gamma\rho}\ C^{aBC}\ f^a_{\gamma\rho}\ \overline{\lambda}^{\dot\alpha\ B}\ \overline{\lambda}^C_{\dot\alpha}$ up to one–loop we find

$$\left(1 - \frac{(N_f - 3)}{(4\pi)^2\ \epsilon} \mathcal{N}\ g^2 \right) g\ \mathcal{F}^{\gamma\rho}\ C^{aBC}\ f^a_{\gamma\rho}\ \overline{\lambda}^{\dot\alpha\ B}\ \overline{\lambda}^C_{\dot\alpha} \tag{C.14}$$

Expressing the renormalized quantities in terms of bare ones we find the condition

$$Z_g^{-1}\ Z_{\mathcal{F}}^{-1}\ Z_{A_a}^{-\frac{1}{2}} \left(1 - \frac{(N_f - 3)}{(4\pi)^2\ \epsilon} \mathcal{N}\ g^2 \right) = 1 \tag{C.15}$$

Inserting the explicit expressions for the renormalization functions as given in (C.8) it is easy to see that this condition is *not* satisfied.

b) We now consider the terms proportional to $g^2 \mathcal{F}^2 (\overline{\lambda\lambda})^2$. From (C.6) we see that these terms receive nontrivial divergent corrections at one loop. However, given the conditions $Z_g Z_{\mathcal{F}} = 1$ and $Z_{\overline{\lambda}} = 1$ we do not have any possibility to cancel these divergences.

Therefore we confirm at the component level the result already evident in superspace, i.e. that the $N = \frac{1}{2}$ SYM theory defined by the action (3.4) as obtained by the natural deformation of the ordinary $N = 1$ SYM theory, is *not* one–loop renormalizable and the classical action has to be extended by adding extra gauge invariant terms.

D Gauge fixing of the modified action

In this Appendix we discuss in detail the gauge–fixing procedure for the modified action (7.1).

In the ordinary anticommutative $SU(\mathcal{N})$ SYM theory described by the action (2.24) the kinetic operator $\mathrm{Tr}\left(V\overline{D}^{\dot\alpha}D^2\overline{D}_{\dot\alpha}V\right)$ is not invertible and one has to implement the gauge-fixing Faddeev–Popov prescription extended to superspace (see for example [26]). In the background field method the standard procedure requires the introduction in the functional integral of quantum gauge–variant but background gauge covariant quantities (the standard choice is $\boldsymbol{\nabla}^2 V$ and $\overline{\boldsymbol{\nabla}}^2 V$) with a suitable weight factor

$$Z = \int \mathcal{D}V\ \mathcal{D}f\ \mathcal{D}\overline{f}\Delta^{-1}(V)\delta[\overline{\boldsymbol{\nabla}}^2 V - f]\delta[\boldsymbol{\nabla}^2 V - f]e^{S_{inv}}\exp\left[-\frac{1}{g^2\alpha}\int d^4x d^4\theta \mathrm{Tr}(f\overline{f})\right] \tag{D.1}$$

where f and $\overline{f}$ are background covariantly (anti)chiral functions and the factor $\Delta^{-1}(V)$ is the ghost action.

Performing the f, $\overline{f}$ integration one obtains the standard gauge-fixing action

$$S_{GF} = -\frac{1}{g^2\alpha}\int d^4x d^4\theta \mathrm{Tr}\left(\overline{\boldsymbol{\nabla}}^2 V\ \boldsymbol{\nabla}^2 V\right) \tag{D.2}$$

which combined with the original kinetic term gives rise to the invertible operator

$$\frac{1}{2\ g^2}\ V^a\ \left[\overline{D}^{\dot\alpha}D^2\overline{D}_{\dot\alpha} - \alpha^{-1}\left(D^2\overline{D}^2 + \overline{D}^2 D^2\right)\right]\ V^a \tag{D.3}$$

The propagator then reads

$$\langle V\ V\rangle = \frac{g^2\ \alpha}{\Box}\ \left[1 + (1-\alpha^{-1})\frac{\overline{D}^{\dot\alpha}D^2\overline{D}_{\dot\alpha}}{\Box}\right] \tag{D.4}$$

and in the Feynman gauge ($\alpha = 1$) we recover the standard vector propagator (3.7).

We now consider, still in the ordinary anticommutative case, the $U(\mathcal{N})$ SYM theory described by the action

$$\frac{1}{2g^2}\int d^4x d^2\overline{\theta}\ \mathrm{Tr}(\overline{W}^{\dot\alpha}\overline{W}_{\dot\alpha})\ +\ \frac{1}{2g_0^2\mathcal{N}}\int d^4x d^2\overline{\theta}\ \mathrm{Tr}(\overline{W}^{\dot\alpha})\mathrm{Tr}(\overline{W}_{\dot\alpha}) \tag{D.5}$$

Including the second term amounts to assigning different coupling constants to the $SU(\mathcal{N})$ and $U(1)$ gauge fields. The kinetic terms for the different components are in fact

$$\frac{1}{2\ g^2}\ V^a\ \overline{D}^{\dot\alpha}D^2\overline{D}_{\dot\alpha}V^a + \frac{1}{2}\left(\frac{1}{g^2} + \frac{1}{g_0^2}\right)V^0\ \overline{D}^{\dot\alpha}D^2\overline{D}_{\dot\alpha}V^0 \tag{D.6}$$

where the label a runs over the $SU(\mathcal{N})$ indices.

In order to fix the gauge, a natural way to proceed is to change the weight factor in (D.1) by adapting it to the different normalizations of the kinetic terms for the abelian and nonabelian gauge fields. This amounts to choosing the weight factor

$$\exp\left[-\frac{1}{g^2\alpha}\int d^4xd^4\theta \mathrm{Tr}(f\overline{f})\right]\times\exp\left[-\frac{1}{g_0^2\mathcal{N}\alpha}\int d^4xd^4\theta \mathrm{Tr}(f)\mathrm{Tr}(\overline{f})\right] \tag{D.7}$$

which leads to the convenient propagators (in Feynman gauge)

$$\langle V^a\ V^b\rangle = \frac{g^2}{\Box}\delta^{ab} \tag{D.8}$$

$$\langle V^0\ V^0\rangle = \frac{\widetilde{g}^2}{\Box} \tag{D.9}$$

with

$$\widetilde{g}^2 = \frac{g^2 g_0^2}{g_0^2+g^2} \tag{D.10}$$

An alternative possibility would be to keep the weight factor as in (D.1). In Feynman gauge this would lead to the propagator (D.8) for the $SU(\mathcal{N})$ fields, whereas the abelian propagator would necessarily have the more general structure

$$\langle V^0\ V^0\rangle = \frac{g^2}{\Box}\left[1+\left(\frac{g^2}{g_0^2+g^2}\right)\frac{\overline{D}^{\dot{\alpha}}D^2\overline{D}_{\dot{\alpha}}}{\Box}\right] \tag{D.11}$$

In the ordinary nonanticommutative case the two choices are equivalent and selecting one or the other is simply a matter of convenience. In the NAC case, instead, because of the presence of the $*$-product, the choice of the weight factor (D.7) is somewhat inconvenient. In fact, after functional integration in f and $\overline{f}$, we would end up with an extra gauge-fixing term in the action

$$-\frac{1}{g_0^2\mathcal{N}\alpha}\int d^4xd^4\theta \mathrm{Tr}(\overline{\boldsymbol{\nabla}}^2V)\mathrm{Tr}(\boldsymbol{\nabla}^2V) \tag{D.12}$$

which is *not* background gauge invariant. Since the power of the background field method lies on this invariance we would be forced to add to (D.7) a suitable completion dependent on background gauge fields in a way very similar to what has been done in (2.22). This would also change the Nielsen–Kallosh ghosts action.

Therefore in this situation it is more convenient to start with the original weight factor (D.1) which does not spoil the background gauge invariance of the gauge–fixing action. The invertible kinetic terms are then

$$\frac{1}{2\ g^2}\ V^a\ \left[\overline{D}^{\dot{\alpha}}D^2\overline{D}_{\dot{\alpha}}-\alpha^{-1}\left(D^2\overline{D}^2+\overline{D}^2D^2\right)\right]\ V^a \tag{D.13}$$

$$\frac{1}{2\ \widetilde{g}^2}\ V^0\ \left[\overline{D}^{\dot{\alpha}}D^2\overline{D}_{\dot{\alpha}}-\widetilde{\alpha}^{-1}\left(D^2\overline{D}^2+\overline{D}^2D^2\right)\right]\ V^0 \tag{D.14}$$

with $\widetilde{g}^2$ as in (D.10) and $\widetilde{\alpha}\equiv\alpha\ \frac{g^2}{\widetilde{g}^2}$. Choosing the Feynman gauge ($\alpha=1$) we finally obtain the propagators (D.8) for $SU(\mathcal{N})$ and (D.11) for the $U(1)$ superfields.

E Background field expansion of the new vertices

In this Appendix we perform the background field expansion of the new vertices emerging from the modified action (7.1). In particular, we extract the terms quadratic in the quantum V field necessary for one–loop calculations.

In antichiral representation the quantum-background splitting reads

$$\overline{\nabla}^{\dot\alpha} = e_*^V * \overline{\boldsymbol{\nabla}}^{\dot\alpha} * e_*^{-V} \equiv \overline{D}^{\dot\alpha} - i\overline{\Gamma}^{\dot\alpha} \qquad\qquad \nabla^\alpha = \boldsymbol{\nabla}^\alpha \tag{E.1}$$

Therefore, expanding up to second order in the quantum V superfields, we find

$$\begin{aligned}\overline{\Gamma}^{\dot\alpha} &\to \overline{\boldsymbol{\Gamma}}^{\dot\alpha} - i\left[\overline{\boldsymbol{\nabla}}^{\dot\alpha}, V\right]_* + \frac{i}{2}\left[\left[\overline{\boldsymbol{\nabla}}^{\dot\alpha}, V\right]_*, V\right]_* \\ &= \overline{\boldsymbol{\Gamma}}^{\dot\alpha} - i\overline{D}^{\dot\alpha}V + \left[V, \overline{\boldsymbol{\Gamma}}^{\dot\alpha}\right]_* - \frac{i}{2}\left[V, \overline{D}^{\dot\alpha}V\right]_* + \frac{1}{2}\left[V, \left[V, \overline{\boldsymbol{\Gamma}}^{\dot\alpha}\right]_*\right]_*\end{aligned} \tag{E.2}$$

The correct expressions for $\overline{W}^{\dot\alpha}$ and $\overline{\Gamma}^{\alpha\dot\alpha}$ follow from the definitions

$$\overline{W}^{\dot\alpha} = D^2\overline{\Gamma}^{\dot\alpha} \qquad\qquad \overline{\Gamma}^{\alpha\dot\alpha} = -iD^\alpha\overline{\Gamma}^{\dot\alpha} \tag{E.3}$$

We are now ready to list the contributions quadratic in V from each new vertex and obtain the new Feynman rules suitable for one-loop calculations. We use conventions introduced in Section 4 and Appendices A and B. In particular, the color tensors $\mathcal{P}$ and $\mathcal{Q}$ are defined in (4.6, 4.7). Moreover we avoid "bold" notation for background superfields.

- " 2-point function – double trace term"

$$\begin{aligned}&\frac{1}{2\, g_0^2\, \mathcal{N}} \int d^4x\, d^4\theta \,\mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\right) \mathrm{Tr}\left(\overline{W}_{\dot\alpha}\right) \to \\ &- \frac{1}{g_0^2\, \mathcal{N}} \int \Big\{ 2i\,\sqrt{\mathcal{N}}\, \sinh\left(\pi_1\wedge\pi_2\right)\, \delta^{AB}\, \delta^{C0}\, V^A(1)\, \overline{\Gamma}^{\dot\alpha B}(2)\, \left(\widetilde{D}^2\widetilde{\overline{D}}_{\dot\alpha}V^C(3)\right) \\ &\qquad\qquad + i\,\sqrt{\mathcal{N}}\, \sinh\left(\pi_1\wedge\pi_2\right)\, \delta^{AB}\, \delta^{C0}\, V^A(1)\, \left(\widetilde{\overline{D}}^{\dot\alpha}\, V^B(2)\right)\, \overline{W}^C_{\dot\alpha}(3)\Big\} \\ &+ \frac{1}{g_0^2\, \mathcal{N}} \int \Big\{ 2\,\sinh\left(\pi_1\wedge\pi_2\right)\sinh\left(\pi_3\wedge\pi_4\right)\, \delta^{AB}\, \delta^{CD}\, V^A(1)\, \overline{\Gamma}^{\dot\alpha B}(2)\, \widetilde{D}^2\left(V^C(3)\, \overline{\Gamma}^D_{\dot\alpha}(4)\right) \\ &\qquad + \sqrt{\mathcal{N}}\, \sinh\left(\pi_1\wedge\pi_2 + \pi_1\wedge\pi_3\right)\delta^{D0}\, \mathcal{P}^{ABC}(\pi_2,\pi_3)V^A(1)\, V^B(2)\, \overline{\Gamma}^{\dot\alpha C}(3)\, \overline{W}^D_{\dot\alpha}(4)\Big\}\end{aligned} \tag{E.4}$$

- " 3-point function"

$$\frac{2\, i\, \mathcal{F}^{\rho\gamma}}{g_0^2\, \mathcal{N}} \int d^4x\, d^4\theta\;\, \overline{\theta}^2\, \mathrm{Tr}\left(\partial_{\rho\dot\rho}\overline{\Gamma}^{\dot\alpha}\right) \mathrm{Tr}\left(\overline{W}_{\dot\alpha}\overline{\Gamma}_\gamma{}^{\dot\rho}\right) \to$$

$$
\begin{aligned}
&- \frac{2\,\mathcal{F}^{\rho\gamma}}{g_0^2\,\mathcal{N}} \int \Big\{ + i\,\sqrt{\mathcal{N}}\;\delta^{A0}\;\delta^{BC}\;\Big(p_{1\rho\dot\rho}\;\overline{\eth}^2\;\overline{\Gamma}^{\dot\alpha A}(1)\Big)\Big(\widetilde{D}^2\widetilde{\overline{D}}_{\dot\alpha}V^B(2)\Big)\Big(\widetilde{D}_\gamma\widetilde{\overline{D}}^{\dot\rho}\,V^C(3)\Big) \\
&\quad - \sqrt{\mathcal{N}}\;\delta^{A0}\;\delta^{BC}\;\Big(p_{1\rho\dot\rho}\;\overline{\eth}^2\;\widetilde{\overline{D}}^{\dot\alpha}\,V^A(1)\Big)\Big(\widetilde{D}^2\widetilde{\overline{D}}_{\dot\alpha}V^B(2)\Big)\;\overline{\Gamma}_\gamma^{\;\dot\rho C}(3) \\
&\quad + i\,\sqrt{\mathcal{N}}\;\delta^{A0}\;\delta^{BC}\;\Big(p_{1\rho\dot\rho}\;\overline{\eth}^2\;\widetilde{\overline{D}}^{\dot\alpha}\,V^A(1)\Big)\;\overline{W}^B_{\dot\alpha}(2)\;\Big(\widetilde{D}_\gamma\widetilde{\overline{D}}^{\dot\rho}\,V^C(3)\Big)\Big\} \\
&- \frac{2\,\mathcal{F}^{\rho\gamma}}{g_0^2\,\mathcal{N}} \int \Big\{ - \frac{1}{2}\,\sqrt{\mathcal{N}}\;\delta^{A0}\;f^{BCD}\;\Big(p_{1\rho\dot\rho}\;\overline{\eth}^2\;\overline{\Gamma}^{\dot\alpha A}(1)\Big)\;\widetilde{D}^2\Big(\widetilde{\overline{D}}_{\dot\alpha}V^B(2)\;V^C(3)\Big)\;\overline{\Gamma}_\gamma^{\;\dot\rho D}(4) \\
&\quad + i\,\sqrt{\mathcal{N}}\;\delta^{A0}\;f^{BCD}\;\Big(p_{1\rho\dot\rho}\;\overline{\eth}^2\;\overline{\Gamma}^{\dot\alpha A}(1)\Big)\;\widetilde{D}^2\Big(\overline{\Gamma}^B_{\dot\alpha}(2)\;V^C(3)\Big)\Big(\widetilde{D}_\gamma\widetilde{\overline{D}}^{\dot\rho}\,V^D(4)\Big) \\
&\quad - \sqrt{\mathcal{N}}\;\delta^{A0}\;f^{BCD}\;\Big(p_{1\rho\dot\rho}\;\overline{\eth}^2\;\widetilde{\overline{D}}^{\dot\alpha}\,V^A(1)\Big)\;\widetilde{D}^2\Big(\overline{\Gamma}^B_{\dot\alpha}(2)V^C(3)\Big)\;\overline{\Gamma}_\gamma^{\;\dot\rho D}(4) \\
&\quad + \frac{1}{2}\,\sqrt{\mathcal{N}}\;\delta^{A0}\;\mathcal{P}^{BCD}(\pi_3,\pi_4)\Big(p_{1\rho\dot\rho}\;\overline{\eth}^2\;\overline{\Gamma}^{\dot\alpha A}(1)\Big)\;\overline{W}^B_{\dot\alpha}(2)\;\widetilde{D}_\gamma\Big(\widetilde{\overline{D}}^{\dot\rho}\,V^C(3)\;V^D(4)\Big) \\
&\quad + \sqrt{\mathcal{N}}\;\delta^{A0}\;\mathcal{P}^{BCD}(\pi_3,\pi_4)\;\Big(p_{1\rho\dot\rho}\;\overline{\eth}^2\;\overline{\Gamma}^{\dot\alpha A}(1)\Big)\Big(\widetilde{D}^2\widetilde{\overline{D}}_{\dot\alpha}V^B(2)\Big)\;\widetilde{D}_\gamma\Big(\overline{\Gamma}^{\dot\rho C}(3)\;V^D(4)\Big) \\
&\quad + \sqrt{\mathcal{N}}\;\delta^{A0}\;\mathcal{P}^{BCD}(\pi_3,\pi_4)\;\Big(p_{1\rho\dot\rho}\;\overline{\eth}^2\;\widetilde{\overline{D}}^{\dot\alpha}\,V^A(1)\Big)\;\overline{W}^B_{\dot\alpha}(2)\;\widetilde{D}_\gamma\Big(\overline{\Gamma}^{\dot\rho C}(3)\;V^D(4)\Big) \\
&\quad + 2\;i\;\delta^{AB}\;\delta^{CD}\;\sinh(\pi_1\wedge\pi_2)\;(p_{1\rho\dot\rho}+p_{2\rho\dot\rho})\;\overline{\eth}^2\overline{\Gamma}^{\dot\alpha A}(1)\;V^B(2)\;\Big(\widetilde{D}^2\widetilde{\overline{D}}_{\dot\alpha}V^C(3)\Big)\;\overline{\Gamma}_\gamma^{\;\dot\rho D}(4) \\
&\quad + 2\;\delta^{AB}\;\delta^{CD}\;\sinh(\pi_1\wedge\pi_2)\;(p_{1\rho\dot\rho}+p_{2\rho\dot\rho})\;\overline{\eth}^2\overline{\Gamma}^{\dot\alpha A}(1)\;V^B(2)\;\overline{W}^C_{\dot\alpha}(3)\;\Big(\widetilde{D}_\gamma\widetilde{\overline{D}}^{\dot\rho}\,V^D(4)\Big) \\
&\quad + i\;\delta^{AB}\;\delta^{CD}\;\sinh(\pi_1\wedge\pi_2)\;(p_{1\rho\dot\rho}+p_{2\rho\dot\rho})\Big(\;\overline{\eth}^2\;\widetilde{\overline{D}}^{\dot\alpha}\,V^A(1)\Big)\;V^B(2)\;\overline{W}^C_{\dot\alpha}(3)\;\overline{\Gamma}_\gamma^{\;\dot\rho D}(4)\Big\} \\
&- \frac{2\,\mathcal{F}^{\rho\gamma}}{g_0^2\,\mathcal{N}} \int \Big\{ + \frac{i}{2}\,\sqrt{\mathcal{N}}\;\delta^{A0}\;f^{FDE}\;\mathcal{P}^{BCF}(\pi_2,\pi_3)\;\times \\
&\qquad\qquad \times\Big(p_{1\rho\dot\rho}\;\overline{\eth}^2\;\overline{\Gamma}^{\dot\alpha A}(1)\Big)\;\widetilde{D}^2\Big(\overline{\Gamma}^B_{\dot\alpha}(2)\;V^C(3)\;V^D(4)\Big)\;\overline{\Gamma}_\gamma^{\;\dot\rho E}(5) \\
&\quad + \sqrt{\mathcal{N}}\;\delta^{A0}\;f^{BCF}\;\mathcal{P}^{DEF}\;(\pi_4,\pi_5)\;\times \\
&\qquad\qquad \times\Big(p_{1\rho\dot\rho}\;\overline{\eth}^2\;\overline{\Gamma}^{\dot\alpha A}(1)\Big)\;\widetilde{D}^2\Big(\overline{\Gamma}^B_{\dot\alpha}(2)\;V^C(3)\Big)\;\widetilde{D}_\gamma\Big(\overline{\Gamma}^{\dot\rho D}(4)\;V^E(5)\Big) \\
&\quad + 2\;i\;\delta^{AB}\;f^{CDE}\;\sinh(\pi_1\wedge\pi_2)\;\;\times \\
&\qquad\qquad \times(p_{1\rho\dot\rho}+p_{2\rho\dot\rho})\;\overline{\eth}^2\overline{\Gamma}^{\dot\alpha A}(1)\;V^B(2)\;\widetilde{D}^2\Big(\overline{\Gamma}^C_{\dot\alpha}(3)\;V^D(4)\Big)\;\overline{\Gamma}_\gamma^{\;\dot\rho E}(5) \\
&\quad - 2\;i\;\delta^{AB}\;\sinh(\pi_1\wedge\pi_2)\;\mathcal{P}^{CDE}(\pi_4,\pi_5)\;\;\times \\
&\qquad\qquad \times(p_{1\rho\dot\rho}+p_{2\rho\dot\rho})\;\overline{\eth}^2\overline{\Gamma}^{\dot\alpha A}(1)\;V^B(2)\;\overline{W}^C_{\dot\alpha}(3)\;\widetilde{D}_\gamma\Big(\overline{\Gamma}^{\dot\rho D}(4)\;V^E(5)\Big) \\
&\quad + \delta^{DE}\;\sinh(\pi_1\wedge\pi_3+\pi_2\wedge\pi_3)\;\mathcal{P}^{ABC}(\pi_1,\pi_2)\;\times \\
&\qquad\qquad \times\;(p_{1\rho\dot\rho}+p_{2\rho\dot\rho}+p_{3\rho\dot\rho})\;\overline{\eth}^2\overline{\Gamma}^{\dot\alpha A}(1)\;V^B(2)\;V^C(3)\;\overline{W}^D_{\dot\alpha}(4)\;\overline{\Gamma}_\gamma^{\;\dot\rho E}(5) \\
&\quad - \frac{i}{2}\,\sqrt{\mathcal{N}}\;\delta^{A0}\;\mathcal{P}^{BFE}(\pi_3+\pi_4,\pi_5)\;\mathcal{P}^{CDF}(\pi_3,\pi_4)\;\times
\end{aligned}
$$

$$\times \left(p_{1\rho\dot\rho}\ \overline{\eth}^2\ \overline{\Gamma}^{\dot\alpha A}(1)\right) \overline{W}^B_{\dot\alpha}(2)\ \widetilde{D}_\gamma \left(\overline{\Gamma}^{\dot\rho C}(3)\ V^D(4)\ V^E(5)\right) \Big\} \tag{E.5}$$

- " 4-point function – double trace term"

$$\begin{aligned}
&\frac{\mathcal{F}^2}{2\ h^2\ \mathcal{N}} \int d^4x\ d^4\theta\ \ \overline{\theta}^2\ \mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\overline{W}_{\dot\alpha}\right) \mathrm{Tr}\left(\overline{W}^{\dot\beta}\overline{W}_{\dot\beta}\right) \rightarrow \\
&\frac{\mathcal{F}^2}{h^2\ \mathcal{N}} \int \Big\{ \ \delta^{AB}\ \delta^{CD}\ \left(\overline{\eth}^2\ \widetilde{\overline{D}}^{\dot\alpha}\ V^A(1)\right)\ \left(\widetilde{D}^2\widetilde{\overline{D}}_{\dot\alpha}V^B(2)\right)\ \overline{W}^{\dot\beta C}(3)\ \overline{W}^D_{\dot\beta}(4) \\
&\qquad + 2\ \delta^{AB}\ \delta^{CD}\ \left(\overline{\eth}^2\ \widetilde{\overline{D}}^{\dot\alpha}\ V^A(1)\right)\ \overline{W}^B_{\dot\alpha}(2)\ \left(\widetilde{D}^2\widetilde{\overline{D}}^{\dot\beta}V^C(3)\right)\ \overline{W}^D_{\dot\beta}(4)\Big\} \\
&+\frac{\mathcal{F}^2}{h^2\ \mathcal{N}} \int \Big\{ -f^{ABC}\ \delta^{DE}\ \left(\overline{\eth}^2\ V^A(1)\right)\left(\widetilde{\overline{D}}^{\dot\alpha}V^B(2)\right)\ \overline{W}^C_{\dot\alpha}(3)\ \overline{W}^{\dot\beta D}(4)\ \overline{W}^E_{\dot\beta}(5) \\
&\qquad - 2\ f^{ABC}\ \delta^{DE}\ \left(\overline{\eth}^2\ \widetilde{\overline{D}}^{\dot\alpha}\ V^A(1)\right)\ \widetilde{D}^2\left(V^B(2)\overline{\Gamma}^C_{\dot\alpha}(3)\right)\ \overline{W}^{\dot\beta D}(4)\ \overline{W}^E_{\dot\beta}(5) \\
&\qquad - 4\ f^{ABC}\ \delta^{DE}\ \left(\overline{\eth}^2\ V^A(1)\right)\ \overline{\Gamma}^{\dot\alpha B}(2)\ \overline{W}^C_{\dot\alpha}(3)\ \left(\widetilde{D}^2\widetilde{\overline{D}}^{\dot\beta}V^D(4)\right)\ \overline{W}^E_{\dot\beta}(5)\Big\} \\
&+\frac{\mathcal{F}^2}{h^2\ \mathcal{N}} \int \Big\{ \ f^{ABG}\ f^{GCD}\ \delta^{EF}\ \left(\overline{\eth}^2\ V^A(1)\right)\ \overline{\Gamma}^{\dot\alpha B}(2)\ \widetilde{D}^2\left(V^C(3)\overline{\Gamma}^D_{\dot\alpha}(4)\right)\ \overline{W}^{\dot\beta E}(5)\ \overline{W}^F_{\dot\beta}(6) \\
&\qquad + 2\ f^{ABC}\ f^{DEF}\ \left(\overline{\eth}^2\ V^A(1)\right)\ \overline{\Gamma}^{\dot\alpha B}(2)\ \overline{W}^C_{\dot\alpha}(3)\ \widetilde{D}^2\left(V^D(4)\overline{\Gamma}^{\dot\beta E}(5)\right)\ \overline{W}^F_{\dot\beta}(6) \\
&\qquad - i\ f^{DAG}\ \mathcal{P}^{GBC}(\pi_2,\pi_3)\ \delta^{EF}\left(\overline{\eth}^2\ V^A(1)\right)\ V^B(2)\ \overline{\Gamma}^{\dot\alpha C}(3)\ \overline{W}^D_{\dot\alpha}(4)\ \overline{W}^{\dot\beta E}(5)\overline{W}^F_{\dot\beta}(6)\Big\}
\end{aligned} \tag{E.6}$$

- " 4-point function – single trace term"

$$\begin{aligned}
&\frac{\mathcal{F}^2}{l^2} \int d^4\theta\ \overline{\theta}^2\mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\ \overline{W}_{\dot\alpha}\ \overline{W}^{\dot\beta}\ \overline{W}_{\dot\beta}\right) \rightarrow \\
&\frac{\mathcal{F}^2}{l^2} \int \Big\{\ \frac{1}{2}\ d^{ABE}d^{ECD}\ \left(\overline{\eth}^2\ \widetilde{\overline{D}}^{\dot\alpha}V^A(1)\right)\ \left(\widetilde{D}^2\widetilde{\overline{D}}_{\dot\alpha}V^B(2)\right)\ \overline{W}^{\dot\beta C}(3)\ \overline{W}^D_{\dot\beta}(4) \\
&\qquad + d^{ABE}\ d^{ECD}\ \left(\overline{\eth}^2\ \widetilde{\overline{D}}^{\dot\alpha}V^A(1)\right)\ \overline{W}^B_{\dot\alpha}(2)\ \left(\widetilde{D}^2\widetilde{\overline{D}}^{\dot\beta}V^C(3)\right)\ \overline{W}^D_{\dot\beta}(4)\Big\} \\
&+\frac{\mathcal{F}^2}{l^2} \int \Big\{\ -\frac{1}{2}\ f^{ABF}d^{FCG}\ \left(if^{GDE}+d^{GDE}\right)\ \times \\
&\qquad\qquad \left(\overline{\eth}^2\ V^A(1)\right)\ \left(\widetilde{\overline{D}}^{\dot\alpha}V^B(2)\right)\ \overline{W}^C_{\dot\alpha}(3)\ \overline{W}^{\dot\beta D}(4)\ \overline{W}^E_{\dot\beta}(5)
\end{aligned}$$

$$
\begin{aligned}
&- \; d^{AFG} f^{BCF} \; \left(if^{GDE} + d^{GDE}\right) \; \times \\
&\qquad\qquad \left(\overline{\eth}^2 \; \widetilde{D}^2 \widetilde{\overline{D}}^{\dot\alpha} V^A(1) \right) \; V^B(2) \; \overline{\Gamma}^C_{\dot\alpha}(3) \; \overline{W}^{D\dot\beta}(4) \; \overline{W}^E_{\dot\beta}(5) \\
&- 2 \; d^{ABF} f^{CDG} d^{FGE} \; \left(\overline{\eth}^2 \; \widetilde{D}^2 \widetilde{\overline{D}}^{\dot\alpha} V^A(1) \right) \; \overline{W}^B_{\dot\alpha}(2) \; V^C(3) \; \overline{\Gamma}^{\dot\beta D}(4) \; \overline{W}^E_{\dot\beta}(5) \Big\} \\
+ \frac{\mathcal{F}^2}{l^2} \int \Big\{ & \frac{1}{2} \, f^{ABG} \; f^{CDH} \; \left(if^{GHI} + d^{GHI}\right) \left(if^{IEF} + d^{IEF}\right) \; \times \\
&\qquad\qquad \left(\overline{\eth}^2 \; V^A(1)\right) \; \overline{\Gamma}^{\dot\alpha B}(2) \; \widetilde{D}^2 \left(V^C(3) \overline{\Gamma}^D_{\dot\alpha}(4)\right) \; \overline{W}^{\dot\beta E}(5) \; \overline{W}^F_{\dot\beta}(6) \\
&+ \; f^{ABG} d^{GCH} f^{DEI} d^{IFH} \; \times \\
&\qquad\qquad \left(\overline{\eth}^2 \; V^A(1)\right) \; \overline{\Gamma}^{\dot\alpha B}(2) \; \overline{W}^C_{\dot\alpha}(3) \; \widetilde{D}^2 \left(V^D(4) \overline{\Gamma}^{\dot\beta E}(5)\right) \; \overline{W}^F_{\dot\beta}(6) \\
&- \; \frac{i}{2} \; f^{AGH} \mathcal{P}^{BCG}(\pi_2, \pi_3) \; d^{HDI} \; \left(if^{IEF} + d^{IEF}\right) \; \times \\
&\qquad\qquad \left(\overline{\eth}^2 \; V^A(1)\right) \; V^B(2) \; \overline{\Gamma}^{\dot\alpha C}(3) \; \overline{W}^D_{\dot\alpha}(4) \; \overline{W}^{\dot\beta E}(5) \overline{W}^F_{\dot\beta}(6) \Big\}
\end{aligned}
\tag{E.7}
$$

- " 4-point function – triple trace term"

$$
\begin{aligned}
&\frac{\mathcal{F}^2}{r^2} \int d^4\theta \; \overline{\theta}^2 \mathrm{Tr}\left(\overline{\Gamma}^{\dot\alpha}\right) \; \mathrm{Tr}\left(\overline{W}_{\dot\alpha}\right) \; \mathrm{Tr}\left(\overline{W}^{\dot\beta} \; \overline{W}_{\dot\beta}\right) \rightarrow \\
&\frac{\mathcal{F}^2 \, \mathcal{N}}{r^2} \int \Big\{ \left(\delta^{A0}\delta^{B0}\delta^{CD} + \delta^{AB}\delta^{C0}\delta^{D0}\right) \left(\overline{\eth}^2 \; \widetilde{\overline{D}}^{\dot\alpha} V^A(1)\right) \left(\widetilde{D}^2 \widetilde{\overline{D}}_{\dot\alpha} V^B(2)\right) \overline{W}^{\dot\beta C}(3) \; \overline{W}^D_{\dot\beta}(4) \\
&\qquad\qquad +4 \; \delta^{A0}\delta^{B0}\delta^{CD} \; \left(\overline{\eth}^2 \; \widetilde{\overline{D}}^{\dot\alpha} V^A(1)\right) \; \overline{W}^B_{\dot\alpha}(2) \; \left(\widetilde{D}^2 \widetilde{\overline{D}}^{\dot\beta} V^C(3)\right) \; \overline{W}^D_{\dot\beta}(4) \qquad \Big\} \\
&+ \frac{\mathcal{F}^2 \, \mathcal{N}}{r^2} \int \Big\{ \; - \; f^{ABC} \; \delta^{D0} \; \delta^{E0} \; \left(\overline{\eth}^2 \; V^A(1)\right) \left(\widetilde{\overline{D}}^{\dot\alpha} V^B(2)\right) \; \overline{W}^C_{\dot\alpha}(3) \; \overline{W}^{\dot\beta D}(4) \; \overline{W}^E_{\dot\beta}(5) \\
&\qquad\qquad - 2 \; f^{ABC} \; \delta^{D0} \; \delta^{E0} \; \left(\overline{\eth}^2 \; \widetilde{D}^2 \widetilde{\overline{D}}^{\dot\alpha} V^A(1) \right) \; V^B(2) \; \overline{\Gamma}^C_{\dot\alpha}(3) \; \overline{W}^{D\dot\beta}(4) \; \overline{W}^E_{\dot\beta}(5) \\
&\qquad\qquad - 4 \; \delta^{A0} \; \delta^{B0} \; f^{CDE} \; \left(\overline{\eth}^2 \; \widetilde{D}^2 \widetilde{\overline{D}}^{\dot\alpha} V^A(1) \right) \; \overline{W}^B_{\dot\alpha}(2) \; V^C(3) \; \overline{\Gamma}^{\dot\beta D}(4) \; \overline{W}^E_{\dot\beta}(5) \quad \Big\} \\
&+ \frac{\mathcal{F}^2 \, \mathcal{N}}{r^2} \int \Big\{ f^{ABG} f^{GCD} \delta^{E0} \delta^{F0} \left(\overline{\eth}^2 \; V^A(1)\right) \overline{\Gamma}^{\dot\alpha B}(2) \; \widetilde{D}^2 \left(V^C(3) \overline{\Gamma}^D_{\dot\alpha}(4)\right) \overline{W}^{\dot\beta E}(5) \; \overline{W}^F_{\dot\beta}(6) \\
&\qquad\qquad - \; i \; f^{AHD} \mathcal{P}^{BCH}(\pi_2, \pi_3) \delta^{E0} \delta^{F0} \left(\overline{\eth}^2 \; V^A(1)\right) V^B(2) \; \overline{\Gamma}^{\dot\alpha C}(3) \; \overline{W}^D_{\dot\alpha}(4) \; \overline{W}^{\dot\beta E}(5) \overline{W}^F_{\dot\beta}(6) \Big\}
\end{aligned}
\tag{E.8}
$$

These vertices together with the ones reported at the end of Section 4 allow us to compute all the divergent terms which arise at one loop for the theory described by the modified action (7.1).

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